Antenna Aided Interference Mitigation for Cognitive Radio

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Acronyms and abbreviations

16-QAM	16 Quadrature Amplitude Modulation
2G	Second Generation
3G	Third Generation
4G	Fourth Generation
ADSL	Asymmetric Digital Subscriber Line
AWGN	Added White Gaussian Noise
BER	Bit Error Ratio
BS	Base Station
CCDF	Complementary Cumulative Distribution Function
CCI	Co-channel Interference
CDF	Cumulative Distribution Function
CDMA	Code Division Multiple Access
CDMA2000	Code Division Multiple Access 2000
CR	Cognitive Radio
CSG	Closed Subscribe Group
CSI	Channel State Information
DAC	Digital-to-Analog Converter
DFE	Decision-Feedback Equalisation
DL	Downlink
DPC	Dirty Paper Coding
FCC	Federal Communications Commission
FDMA	Frequency Division Multiple Access
GSM	Global System for Mobile communications
i.i.d.	Independent and Identically Distributed
IC	optimal Interference Constraint precoding
IF	optimal Interference Free precoding
IFC	Interference Channel
IF-IF	Interference-Free-and-Interference-Free precoding
kbps	kilobit per second

km	kilometre
LCC	Low Computational Complexity
LTE	Long Term Evolution
M-QAM	M-Quadrature Amplitude Modulation
MAC	Media Access Control
Mbps	Megabit per second
MHz	Megahertz
MIMO	Multiple Input and Multiple Output
MIMO-DFE	Multiple Input and Multiple Output Decision-Feedback Equalisation
MIMO-MUD	Multiple Input and Multiple Output Multi-User Detection
MISO	Multiple Input and Single Output
MISO-CR	Multiple Input and Single Output Cognitive Radio
ML	Maximum Likelihood
MMSE	Minimum Mean Squared Error
MRC	Maximum Ratio Combination
MRT	Maximum Ratio Transmission
MS	Mobile Subscribe
MSE	Mean Squared Error
MSMSE	Minimum Sum Mean Squared Error
MU-CR	Multi-user Cognitive Radio
Ofcom	Office of Communications
OFDM	Orthogonal Frequency-Division Multiplexing
OFDMA	Orthogonal Frequency-Division Multiple Access
PDF	Probability Density Function
QoS	Quality of Service
QPSK	Quadrature Phase-Shift Keying
RF	Radio Frequency
Rx	Receiver
SDR	Software Defined Radio
SER	Symbol Error Ratio
SIC	Successive Interference Cancellation
SIMO	Single Input and Multiple Output
SINR	Signal to Interference and Noise Ratio

SIR	Signal to Interference Ratio
SISO	Single Input and Single Output
SLIR	Signal to Leak Interference Power Ratio
S-Max	Sub-optimal Maximum
S-Min	Sub-optimal Minimum
SMSE	Sum Mean Squared Error
SNR	Signal to Noise power Ratio
SON	Self-Organising Network
SUM-R	Sum Rate
SVD	Singular Value Decomposition
TDD	Time Division Duplex
TDMA	Time Division Multiple Access
TD-SCDMA	Time Division-Synchronous Code Division Multiple Access
THP	Tomlinson-Harashima Precoding
Tx	Transmitter
TxMF	Transmitter Match Filter
TxZF	Transmitter Zero-forcing
UL	Uplink
UMTS	Universal Mobile Telecommunications System
UWB	Ultra-wideband
WCDMA	Wideband Code Division Multiple Access
WiMAX	Worldwide Interoperability for Microwave Access
WLAN	Wireless Local Area Network
ZF	Zero-forcing

Nomenclature

$\mathbf{B}$	feedback matrix in DFE
C	capacity of radio system
C_i	capacity of system i when there is no interference
$CN(m, \sigma^2)$	complex normal distribution with mean m and variance σ^2
$\mathbb{C}$	complex space
$\mathbb{C}^{N_A \times N_B}$	N_A-by-N_B dimensional complex space
$\mathbf{D}$	real diagonal matrix whose non-zero elements are singular of channel matrix
$\mathrm{E}[\cdot]$	mathematical expectation
$\mathrm{E}_1(x)$	order one exponential integral function, $\mathrm{E}_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt$
$F_Z(\cdot)$	complementary cumulative distribution function of random variable Z
$\mathbf{F}$	feed-forward matrix in DFE
$\mathbf{F}_B$	interference channel self-correlation matrix of secondary system
G_i	received data power for system i
G_B^{MRT}	received power for secondary system when MRT is used in secondary system
$\overline{G}$	data channel gain of secondary system when beamforming is used
$\overline{G}_{IF}$	data channel gain of secondary system when IF is used
$\overline{G}_{IC}$	data channel gain of secondary system when IC is used
$\overline{G}_{IF}^j$	data channel gain when antenna subset ω_j is selected and IF is used
$\mathbf{G}$	diagonal matrix in DFE
$\mathbf{G}_B$	date channel self-correlation matrix of secondary system
$\mathbf{H}$	channel matrix
I_i	interference power come from system i
I_B^{MRT}	interference power to the primary system when MRT is used in secondary system
$\overline{I}$	interference channel gain of secondary system when beamforming is used
$\mathbf{I}_{N_R}$	identity matrix with size N_R

J	maximum acceptable interference to primary system
K	number of possible antenna selection (select n antennas from N available antennas, $K = \binom{N}{n}$)
K_1	computational complexity of subset selection one, $K_1 = \binom{N}{t}$
K_2	computational complexity of subset selection two, $K_2 = \binom{N-m}{t-m}$
M	number of points in constellation (M-QAM)
$\mathbf{M}$	transformation matrix used in linear detector or precoding
MSE_i^{nr}	normalised MSE of system i
MMSE_i^{nr}	normalised MMSE of system i
N	number of antennas
N_A	number of antennas for transmitter of primary system
N_B	number of antennas for transmitter of secondary system
N_R	number of antennas in receiver
N_T	number of antennas in transmitter
$N(m, \sigma^2)$	normal distribution with mean m and variance σ^2
P_i	transmission power of system i
Q_i	SLIR value for antenna i
R	throughput
R_{cell}	cell radius in single cell channel
R_i	achievable rate of transmitter receiver i in AWGN interference channel
$R_A^{(Op)}$	throughput of primary system for optimal continuous sum throughput solution
$R_A^{(0)}$	throughput of primary system for round-down discrete sum throughput solution
$R_B^{(Op)}$	throughput of secondary system for optimal continuous sum throughput solution
$R_B^{(0)}$	throughput of secondary system for round-down discrete sum throughput solution
$\mathbb{R}$	real space
$S_1(n, t, N)$	subset optimal selection strategy one with parameter n, t, and N
$S_2(n, m, t, N)$	subset optimal selection strategy two with parameter n, m, t, and N
$\mathbf{S}$	diagonal matrix in DEF

SINR_i	SINR of system i
SINR_{up}	upper bound of SINR for primary system
SINR_{low}	low bound of SINR for primary system
SINR_i^j	SINR of system i when antenna subset ω_j is selected
SNR_i	SNR of system i
$\text{Tr}(\cdot)$	trace of matrix $\cdot$
$\mathbf{U}$	unitary matrix which is used to filter received signal vector in SVD
$\mathbf{V}$	unitary matrix used to precode transmitted signal vector in SVD
a_I	real part of constellation point
a_Q	imaginary part of constellation point
a_{ij}	interference channel power gain from transmitter i to receiver j
d	distance between BS and MS in single cell fading model
$\det(\cdot)$	determinant of matrix $\cdot$
$\text{diag}\{x_1, \cdots, x_n\}$	diagonal matrix with diagonal elements $x_1, \cdots, x_n$
$f_Z(\cdot)$	probability density function of random variable Z
g_i	complex scalar in receiver of system i
$\widehat{g}_i$	complex scalar in receiver of system i, which achieves normalised MMSE
$\mathbf{g}$	channel vector representing Rayleigh fading with zero mean and unit variance
h	channel for the primary system for single antenna transmitter
h_{Di}	ith elements of $\mathbf{h}_D$
h_{Ii}	ith elements of $\mathbf{h}_I$
$\mathbf{h}_{AA}$	data channel vector for primary system
$\mathbf{h}_{BB}$	data channel vector for secondary system
$\mathbf{h}_{AB}$	interference channel vector from primary system to secondary system
$\mathbf{h}_{BA}$	interference channel vector from secondary system to primary system
$\mathbf{h}_D$	data channel vector of secondary system for antenna selection scenario
$\mathbf{h}_I$	interference channel vector of secondary system for antenna selection scenario

$\mathbf{h}^j$	channel vector when antenna subset ω_j is selected
l	sample time
m	number of antennas which have the largest norm and are always selected
$\mathbf{m}_0$	eigenvector corresponding the non-zero eigenvalue of $\mathbf{G}_B$
$\min(x, y)$	minimum value of x and y
n	number of RF chains in the transmitter of the secondary system
r_{AB}	interference channel factor for primary system
r_{BA}	interference channel factor for secondary system
$\mathrm{rank}(\cdot)$	rank of matrix $\cdot$
s_i	transmitted signal of system i
$s_i(l)$	transmitted symbol at sample time l for system i
t	size of the antenna subset selected from the N available antennas
$\mathbf{u}_0$	eigenvector corresponding the non-zero eigenvalue of $\mathbf{F}_B$
$\mathbf{v}_A$	precoding vector for primary system
$\mathbf{v}_B$	precoding vector for secondary system
$\mathbf{v}_B^{IC}$	IC precoding vector for secondary system
$\mathbf{v}_B^{IF}$	IF precoding vector for secondary system
$\mathbf{v}_B^{MRT}$	MRT precoding vector for secondary system
$\mathbf{v}_B^{ZF}$	ZF precoding vector for secondary system
$\mathbf{v}_i^{op}$	optimal precoding vector of system i for overall joint beamforming
w	weight indicating which system is more important in terms of performance optimisation
$\mathbf{x}$	transmitted signal vector
$\overline{\mathbf{x}}$	precoded transmission signal vector
y_i	received signal of system i
$y_i(l)$	received signal at sample time l for system i
$\mathbf{y}$	filtered received signal vector
$\overline{\mathbf{y}}$	received signal vector
z_i	added Gaussian noise in receiver i
$\mathbf{z}$	added Gaussian noise vector
$\overline{\mathbf{z}}$	filtered added Gaussian noise vector

Γ	SNR gap
Γ_A	SNR gap for primary system
Γ_B	SNR gap for secondary system
Φ_{min}	interference channel selection gain
Ψ_{max}	date channel selection gain
Ω	set of all possible antenna selection, $\Omega = \{ \omega_1, \omega_2, \cdots, \omega_K\}$
α	path loss exponent
β	interference coefficient of secondary system
β_i	interference coefficient of system i
β_i^{Ad}	interference coefficient of system i for adaptive beamforming
β_i^{Op}	optimal interference coefficient of system i for overall joint beamforming
γ	Euler constant, 0.5772156649
$\gamma(a, x)$	lower incomplete gamma function, $\gamma(a, x) = \int_0^x t^{a-1}e^(- t)dt$
δ	shadowing in single cell model
$\varepsilon(\beta, \rho)$	function related to data channel gain for given interference coefficient β and cross-correlation coefficient ρ
$\varepsilon_i(\beta)$	function related data channel gain of system i for a given interference coefficient β when cross-correlation coefficient ρ fixed
ζ_j	interference channel gain when antenna j is selected
ζ_{min}	minimum interference channel gain for single antenna selection
η_j	data channel gain when antenna j is selected
η_{max}	maximum data channel gain for single antenna selection
θ	angle between interference channel and data channel
λ_i	ith eigenvalue of matrix
$\lambda(\cdot)$	eigenvalue set of matrix $\cdot$
μ	water lever used in water-filling algorithm
ξ	average SNR of MIMO system
ρ_i	cross-correlation coefficient for system i
ϱ	convergence condition for iterative algorithm
σ_i^2	noise variances of system i
σ_δ^2	variance of shadow fading δ in single cell channel

ς	search precision for iterative algorithm
$\phi(x)$	a function, defined as $\phi(x) = xe^{x}\mathrm{E}_1(x)$
$\chi^2(\cdot)$	chi-square distribution
ω_i	subset of antenna, elements of Ω, where $i = 1, \cdots, K$
ω_{op}	optimal antenna subset
$(\cdot)^H$	conjugate transpose of matrix $\cdot$
$(\cdot)^*$	conjugate of matrix $\cdot$
$(\cdot)^T$	transpose of matrix $\cdot$
$(x)^+$	equal to x if x is greater than 0; otherwise equal to 0
$\|\|\mathbf{h}\|\|$	norm or length of vector $\mathbf{h}$, defined as $(\sum h_i h_i^*)^{1/2}$
$\|x\|$	norm of scalar x, defined as $(xx^*)^{1/2}$
∞	positive infinity

Chapter 1
Introduction

Wireless communications have greatly changed peoples' lives. Nowadays, people can exchange information wherever they are and whenever they want. The development of radio communication not only enhances the freedom of communications, but also enhances the lifestyle of human beings. People can watch television programs when they are on the road, can receive and reply to business messages wherever they are, can talk with someone without having to sit together when they are travelling, etc. All of these prove that the wireless communication has already become a part of human life. In this chapter, we briefly introduce wireless communications and one of its important applications - cellular radio system, then summarise the major contributions of this thesis and outline the structure of this thesis.

1.1 Wireless communications

Wireless communications have been used by human beings for a long time, even before civilisation. Generally, wireless communication is a method which allows people to transmit and receive information without a fixed line. In ancient times, sound, smoke, fire, etc., were used as basic wireless communication tools. However, all of these have very limited transmission distances. If a long distance communication is needed, they have to build and maintain a lot of relays and it will cost lots of resources. Furthermore, these old wireless communication methods usually can not carry much information due to their simple styles. Wireless communications over long distances and using complex messages were impossible in that time.

This changed after the discovery of electromagnetic waves in the end of the 19th century. People found that the electromagnetic waves can travel a long distance and can be detected by suitable equipment, often called a receiver. Lots of research and experiments have been done in order to implement the wireless communication using electromagnetic waves. In 1895, Marconi demonstrated the first radio transmission between two distant places and this can be seen as the birth of radio communications. After that, radio communication advanced quickly to enable transmission over longer distance, using lower power consumptions, higher throughput, more reliable communications, cheap devices, etc. All of these make radio communications can not only be used by governments, business, and organisations, but also can be privately used by individuals. Wireless communications are used in all kinds of fields of life and lots of new applications are advanced everyday. Currently the most popular applications of wireless communications may include mobile networks, wireless local area networks (WLAN), and television broadcast networks. Televisions are almost universal electronic equipment in families and it also is an important form of relaxation for human beings. The mobile networks and wireless local area networks have not only become the critical parts of business communications but also one of the key methods to form and develop personal relationships. People have already developed powerful mobile networks and they will be introduced in the next section.

1.2 Cellular radio system

Cellular radio system is one of most important applications of radio communications. Currently there are more than two billion users of cellular radio systems and cellular networks are distributed almost all over the world. The first successful and widely accepted cellular network is GSM (Global System for Mobile Communications) systems which have been deployed worldwide as second-generation (2G) mobile communications systems. It is originally developed to transmit voice but also can be used for data transmission. It can support maximum 9.6kbps for uplink and downlink. In order to satisfy the throughput requirements of new applications, the third-generation (3G) mobile communication systems are developed by Europe, China, and USA respectively. There are three kinds of 3G mobile systems, including WCDMA (wideband code division multiple access), TD-SCDMA (time division - synchronous code division multiple access), and CDMA2000 (code division multiple access 2000). All of these are based on CDMA (code division multiple access) technology and can greatly increase the throughput compared to 2G mobile systems. The WCDMA system uses a 5MHz bandwidth and originally the peak data rate was 2Mbps. The updated version of WCDMA system can support up to 28Mbps for downlink and 11Mbps for uplink. The CDMA2000 system only uses 1.25MHz of spectrum but its maximal supported data rate is 307.7kbps which is much smaller compared to the WCDMA system. The TD-SCDMA system is the combination of a TDMA (time division multiple access) component and a CDMA component. This system uses of 1.6MHz spectrum and can support up to 2Mbps for the initial version and 2.8Mbps for the updated version. To further increase the throughput, the fourth generation (4G) mobile network also has been studied. Currently, there are two technologies that can be seen as potential 4G technology: one is LTE (Universal Mobile Telecommunications System Long Term Evolution) and the other is WiMAX (Worldwide Interoperability for Microwave Access). Unlike the 3G mobile networks, OFDM (orthogonal frequency-division multiplexing) not CDMA technology is used in both LTE and WiMAX. The multiple antenna technologies will also be used in these two networks and both of the radio networks will support up to more than 100Mbps data rate.

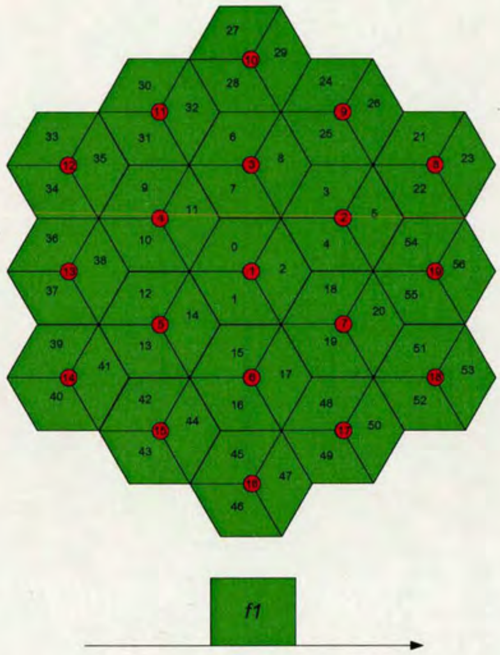


Figure 1.1: *Example of cell deployment: reuse one with three sectors*

From the above description of mobile radio networks, it is clear to see that the main purposes of study on mobile radio network are to support more applications according to the requirements, increase the cell throughput with limited spectrum resource, and improve the reliability of wireless communications. However, one of major factors to limit the cell throughput and coverage of mobile radio network is the co-channel interference (CCI) caused by reusing of spectrum. Since the total spectrum is limited, some cells have to use the overlapping spectrum resource and this will reduce the performance of mobile network. Figure 1.1 and Figure 1.2 show two examples of cell planning for cellular systems, one is reuse one case and the other is reuse three case. In the two examples, each cellular network includes 19 cells. The base stations (BS) in each cell uses directional antennas and a cell is divided to three sectors. In Figure 1.1, every

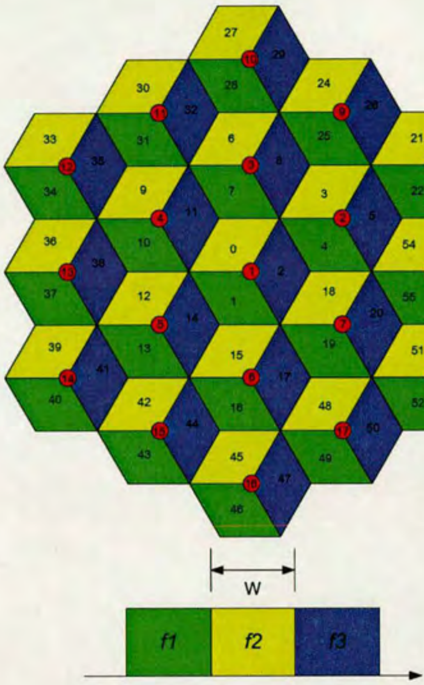


Figure 1.2: *Example of cell deployment: reuse three with three sectors*

sector use the same spectrum and thus it will receive the interference from other cells and other sectors in its own cell. In Figure 1.2, the sectors in one cell use different spectrum resource but every cell uses the same spectrum. In this case one sector in a cell only receive the interference from other cells and there is no intra-cell interference. Of course the reuse three case will have a better coverage and a higher cell throughput due to less interference, but it needs three times more spectrum compared to the reuse one scenario. Therefore, one of the major challenges of a cellular radio system is how to control the co-channel interference to enable low spectrum reuse factors when spectrum resources are scarce. In this thesis we consider interference mitigation using multiple antennas when multiple radio systems coexist in overlapping spectrum.

1.3 Contributions and structure of thesis

The major objective of this thesis is applying multiple antenna technology to coexisting radio environments to cancel or control the co-channel interference and improve the performance of coexisting radio systems. We need to show how the multiple antenna technologies are used in cognitive radio environments and what performance the cognitive radio system can achieve with the help of multiple antennas while considering the computational complexity and the hardware costs. Basically, we will focus our interest on downlink multiple input and single output cognitive radio environments.

This thesis is organised in six chapters and in the introduction chapter a general view of wireless communications are given. We also describes the basic idea of cellular radio system which is a major research topic and coexistence of cellular networks may be an important application of cognitive radio. The rest of this thesis is organised as follows:

Chapter 2 describes the background information of this thesis. We firstly discuss the concept of cognitive radio, then analyse the interference scenarios and the possible applications. Furthermore the basic interference channel model and fundamental results of the interference channel are presented. An introduction to multiple antenna technologies forms the final part of this chapter.

Chapter 3 discusses the single system precoding method for cognitive radio environments. An optimal interference constrained precoding method is proposed for a given interference limit of the primary system when only interference channel and data channel state information are known by the transmitter of the secondary system. This chapter also compares the performance of several single system precoding methods and it shows that the proposed method achieves the best performance with the constraint of a given interference power level.

Chapter 4 discusses and analyses the joint beamforming approaches when both the interference channel and data channel information of the primary system and the secondary system are known by the two transmitters. The adaptive joint beamforming and overall performance optimisation beamforming methods are proposed and analysed in this scenario. The adaptive joint beamforming can dynamically adjust the precoding vectors according to the channel state information, and it maximises the signal to interference and noise ratio of the secondary system while keeping the QoS (quality of service) of the primary system at a certain level. For the overall performance optimisation beamforming, the sum mean squared error and sum throughput are used as our optimisation metrics.

Chapter 5 applies antenna selection technology in multiple input and single output cognitive radio environment to reduce the cost of the radio chain because of the application of multiple antennas. Two scenarios, single antenna selection and multiple antenna selection with beamforming, are considered. For the single antenna selection scheme, a maximum signal to leak interference power ratio method is proposed to achieve a performance trade-off between the primary system and the secondary system. A subset optimal selection method is proposed for the multiple antennas selection scenario to reduce the computational complexity while keeping near optimal performance.

Chapter 6 summarises and concludes this thesis. The limitations of this thesis and possible further research topics for cognitive radio are also presented in this chapter.

Chapter 2
Cognitive Radio and Multiple Antennas

The interest of this thesis is to discuss and design advanced transmitter and/or receiver techniques for coexisting radio environments. More specifically this thesis deals with the application of multiple antenna technologies in cognitive radio systems to increase the overall spectrum efficiency with reasonable and acceptable effects on the radio network that owns licensed spectrum.

The background of this thesis is introduced in this chapter. In Section 2.1, the basic ideas of cognitive radio, its benefits and technical difficulties, interference scenarios, and possible applications are first reviewed; then for better understanding the physical layer model of cognitive radio, a brief description of the interference channel, which has already been studied for more than thirty years, is given in Section 2.2; Section 2.3 is an introduction of multiple antenna technologies, and finally this chapter is concluded in Section 2.4.

2.1 Cognitive radio

The cognitive radio is an novel idea, which comes from Mitola [3], to increase overall spectrum efficiency. In this section, the background of cognitive radio is reviewed. Basically this section answers the following three questions:

1. Why and what is cognitive radio, including the benefits and technical challenges?
2. What kind of interference do cognitive radio systems have to suffer?
3. How can the cognitive radio technologies be used?

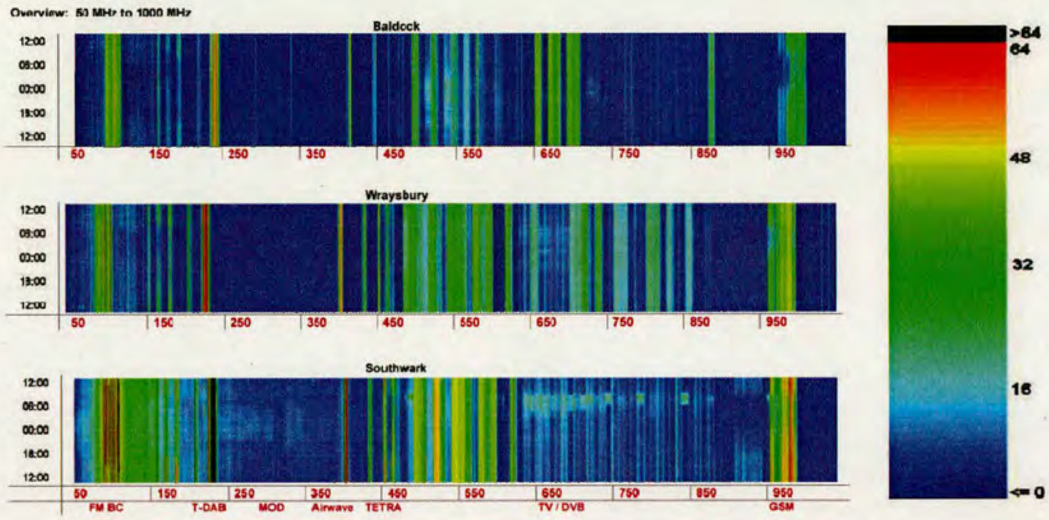


Figure 2.1: *Measurements of spectrum usage near London (Ofcom)*

2.1.1 Introduction

The development of radio communication technologies have greatly increased people's living quality. Nowadays, many wireless communication applications have emerged and people require wireless communications to have higher transmission rates, higher reliability, and better coverage. Thus the frequency spectrum has become more and more congested. Some investigations, however, illustrate that the spectrum resources allocated to users are often not used sufficiently [4] [5]. For example, the Office of Communications (Ofcom) has made measurements of spectrum usage near London in different places and times, and the results are shown in Figure 2.1[1] [6]. This result clearly indicates that many spectrum resources are not fully utilised. Only a small percentage of spectrum is used in any time. This gives an opportunity to increase spectrum efficiency and expand radio access via reusing the spectrum by other radio communication systems when it is vacant. Cooperation between two radio systems may or may not be needed depending on scenarios. This idea, called "cognitive radio" and advanced by Joe Mitola in his PhD thesis in 2000 [3], has attracted lots of attention from academia and industry.

[1]There is no clear units for the usage of spectrum in Figure 2.1 on Ofcom website. Basically the lower value indicates the lower spectrum usage and higher value means the higher spectrum usage in this figure [6].

2.1.1.1 Definition of cognitive radio

Cognitive radio can be seen as an adaptive software defined radio (SDR) according to the radio frequency (RF) environment. But what is the essence of cognitive radio? A generic definition of cognitive radio is proposed by Simon Haykin [7]:

> "The cognitive radio is an intelligent wireless communication system, which can be adaptive to its environment through adjusting its internal states in order to support highly reliable communications and efficient utilisation of the radio spectrum."

This definition expresses at least three important requirements:

1. In cognitive radio environments radio systems are coexisting. This is why the radio systems need to sense the radio environment. The word "coexistence" means that several systems operate in an overlapping frequency domain.
2. Radio systems need to adjust their internal states to increase their performance, such as spectrum efficiency and quality of communication (e.g.,bit error rate). These internal states may include transmission power, carrier frequency, modulation constellation, transmission and detection strategy, and so on.
3. Cognitive radio systems are intelligence radio systems. It can adaptively adjust its state according to surrounding radio environments and its own requirements, such as interference level, spectrum usages, and throughput requirement.

Altogether, the key point of cognitive radio is "spectrum sharing". Basically, two or more radio systems utilising the overlapping RF band can be included in cognitive environments. Some of them are licensed primary systems which have higher priority and the others are secondary systems with low priority. Higher priority systems have higher communication quality guarantees or preferential access to the radio resource. The secondary system can access the licensed spectrum only when it causes no interference or acceptable interference to the primary system. However, if the legacy radio systems are directly used as the primary system in cognitive radio environment, the performance improvement, including spectrum efficiency, coverage, reliability, etc., may be very limited. One reason is these radio systems usually have very limited capability to counter interference, especially from coexisting radio systems, and some radio systems even are not aware of the existence of coexisting radio system because they can not distinguish the coexisting interference from received interference and noise. The other reason is current radio systems have no capability to measure interference channels to/from coexisting radio system, and this information usually is very important for interference management for cognitive radio. Without loss of generality, a typical cognitive radio scenario which will be discussed in the following part of this thesis is presented in Figure 2.2. The primary system is denoted as System A, and the secondary system is denoted as System B.

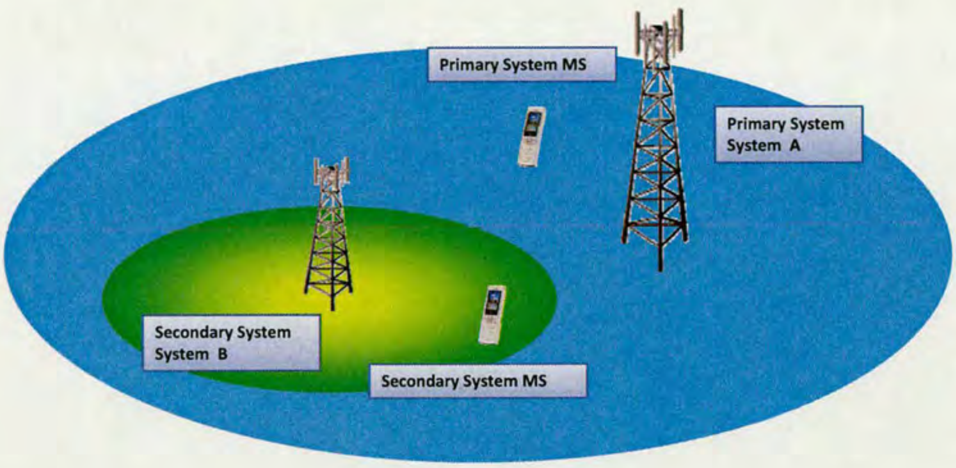


Figure 2.2: *Typical scenario of cognitive radio*

Some papers that discuss coexisting scenarios are now considered. The coexisting scenarios are classified into three types based on the discussion in [8], including underlay, overlay, and interweave approaches. The underlay system would allow the primary system and the secondary system to transmit signals at the same time, and the secondary system uses the spectrum mask to guarantee that the interference generated by the secondary system is below the acceptable interference level for the primary system. Typically, it is a wideband system coexisting with a narrow band system, and the narrow band system appears as co-channel interference to the wideband system. Several papers have focused on such a coexisting scenario, such as ultra-wide band (UWB) [9] [10] [11], or code division multiple access system (CDMA) coexisting with other relatively narrow band radio systems [12]. The overlay system also allows the co-existing radio system to transmit signals while the primary system is using the spectrum. The secondary system only uses part of its power to transmit its own signals and the rest of the power is used to help the primary system to cancel the co-channel interference. Normally, the secondary system needs to know the signals transmitted by the primary system, and relay technology [13] [14] [15] or dirty paper coding [16] [17] technology may be used in such scenarios. The last one is the interweave approach, in which the secondary system only transmit its signals when it finds a "hole". This hole may be in the time domain [18], the frequency domain [19] [20] or any orthogonal domain, and it may change according to the primary radio. The secondary system dynamically detects these holes and utilises them without generating interference to the primary system.

2.1.1.2 Benefits and technical challenges of cognitive radio

The benefits of cognitive radio is as follows:

Spectrum efficiency One of the major benefits of cognitive radio is to improve the spectrum efficiency. It allows multiple radio systems to share the spectrum in order to improve the system throughput. Some papers have already discussed the throughput issues for cognitive radio. Reference [8] discussed the potential throughput of cognitive radio for different cognitive radio scenarios; the theoretical capacity of cognitive radio when side information is available is presented in [16] and [17].

Improve quality of service (QoS) For individual users, the cognitive radio can improve their quality of service. Firstly, the total throughput will increase, and it means each user can have a higher traffic rate or more users can be supported. Moreover, the users can access the radio network via the secondary system when the signal of the primary system is too weak to be received because of channel fading, and it means the reliability and coverage of communication are improved.

Higher power efficiency The cognitive radio system can improve the power efficiency since normally it should have a good power control in order to reduce the interference to the primary system. For the same reason, the coexisting radio system only covers the necessary communication area which usually is a small area. All of these can reduce the power consumption and supply high throughput and therefore increase the power efficiency. It is also positive for the environment, and a similar concept, green radio [21], has been proposed recently to save power and decrease the emission of carbon dioxide.

Lower spectrum costs For the point of view of operators, the cost of spectrum will be reduced when using cognitive radio. One operator can share the spectrum with other operators when it does not fully use the spectrum, therefore these operators who are coexisting in overlapping spectrum can share the costs and revenues of using the spectrum. This also gives regulator flexibility of allocating spectrum resource.

Despite the above benefits, cognitive radio researchers have to solve the following technical challenges in order to apply this idea in the real world:

Co-channel interference Since these radio systems utilise overlapping frequency space, they would have to suffer co-channel interference from each other whichever type of coexistence is used. As per the definition of cognitive radio, the secondary system should not generate serious interference to the primary system and only a little degradation on the performance for the primary system can be acceptable. Thus, how to eliminate the interference is the first challenge for cognitive radio systems.

Extra information In order to control and manage the interference, the coexisting radio system needs to know some extra information about surrounding environments and even some information from the primary system. For example, if the secondary system would like to use the spectrum which is not used by the primary system, it must firstly sense the radio environments and detect which spectrum can be used. This procedure has to be done repeatedly since the primary system may change the spectrum allocation from time to time. Moreover, it will be good for the secondary system if it knows the interference channel between the base station of the secondary system and the terminals of the primary system. This information may be used to determine the maximum transmission power of the secondary system. However, current radio systems do not provide such information. This means current radio systems can not be used directly in cognitive radio environments and modifications must be implemented not only for the secondary system but also for the primary system.

Complexity The cognitive radio system increases the spectrum efficiency and QoS of users at the cost of extra complexity. This may include both hardware complexity and software complexity. Some new equipment may be needed to sense the radio environment and mitigate interference. Since cognitive radio has to be adaptive according to the environment, dynamic algorithms also have to be used. All of these will increase the complexity of a cognitive radio system.

2.1.1.3 Key features of cognitive radio

Cognitive radio is a very complex technology, and it may include many techniques from the physical layer to higher layers. A brief discussion about three mainly research topics in physical layer and MAC layer for cognitive is listed in the following:

Spectrum sensing is an important functionality in cognitive radio since the secondary system needs to know how the spectrum is used by the primary system. Generally, there are three types of spectrum sensing, and they are energy detection [22], coherent detection [23], and cyclostationary feature detection [23]. These detection methods may be used in different scenarios. Energy detection is the optimal approach if only the local noise power is known by the secondary system [24]. However if the secondary system knows some information of the primary system, such as the pilots, preamble, or any synchronisation message formats, it will be much better to use coherent detection. The cyclostationary detector is used to obtain the power of the primary system from received interference and added noise.

Interference mitigation is an key issue for cognitive radio since it directly links to the performance of the cognitive radio network. Multiple user detection and multiple antenna technologies can be used for interference mitigation in cognitive radio. Since multiple antennas can supply extra degrees of freedom for radio systems and have the ability to control interference, lots of papers apply multiple antennas in cognitive radio networks [25] [26] [27] [28] [29]. Generally, multiple antennas, combined with other techniques, are used to achieve the best trade-off between the primary system and the secondary system according to difference scenarios.

Resource allocation is another important topic for cognitive radio. The resource may include spectrum, time slots, transmit power, etc. The secondary system can control or avoid interference to the primary system through allocating user different resource according to surrounding radio environments.

2.1.2 Analysis of interference scenarios

As was discussed in the previous subsection, the key problem of cognitive radio is co-channel interference. In this subsection, we analyse the interference scenarios for cellular cognitive radio systems to see how the coexistence impacts on the performance of radio systems. In this subsection, we assume that there are two cellular radio systems and both radio systems are time division duplex (TDD) systems, as shown in Figure 2.3 (Note: both radio system are multiple cell cellular systems, and only one cell is presented in this figure.)

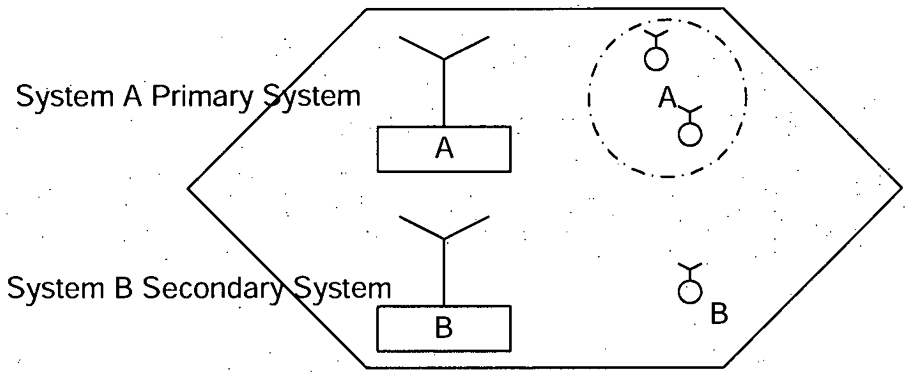


Figure 2.3: *Coexistence of two cellular radio systems*

Firstly, we assume that the two radio systems have the same frame structure and they are synchronised, and the two systems switch between uplink and downlink at the same time, as in Figure 2.4. Then for the downlink of the primary system, the interference received by the user of system A includes two parts: one is from the base stations of its own system which use the same spectrum resource as this cell (inter-cell interference), and the other is from the base stations of the secondary radio system which share the same spectrum resource (inter-radio interference). The uplink interference is similar with the downlink case. The interference received by the base stations of the primary system in a cell also includes two parts: one is from the users of other sector/cell in its own network, and the other is from the users of the coexisting radio network. All the users who cause interference to this cell users share or reuse the spectrum which is used by this cell.

Now we consider more complex scenario in which the two radio system are unsynchronised, shown in Figure 2.5. In this case, the downlink interference received by users of the primary system not only include the interference come from the base stations (the base stations in its own system who reuse the spectrum and the base stations from the coexisting radio system who share the spectrum), but also includes the users of the coexisting radio system who send uplink data to its serving base stations. Furthermore, it is more serious when the user who generates the interference is near to the user who tries to receive signals from its serving base station.

Figure 2.4: *Interference scenario of synchronised CR*

Figure 2.5: *Interference scenario of unsynchronised CR*

For the uplink scenario, the base station will not only be interfered with by other co-channel users but also the base stations of the coexisting radio system who also utilise the overlapping spectrum resource since the two coexisting radio systems are not synchronised.

In the rest of this thesis, we will only consider the synchronised scenario in order to simplify the problem.

2.1.3 Application of cognitive radio

Basically, cognitive radio technologies can be used in any environment in which multiple radio systems share the spectrum and need to dynamically control and mitigate the interference with some intelligence. In this subsection, two very hot topics, femtocell and self-organising networks (SON), in which cognitive radio technologies may be used, are introduced.

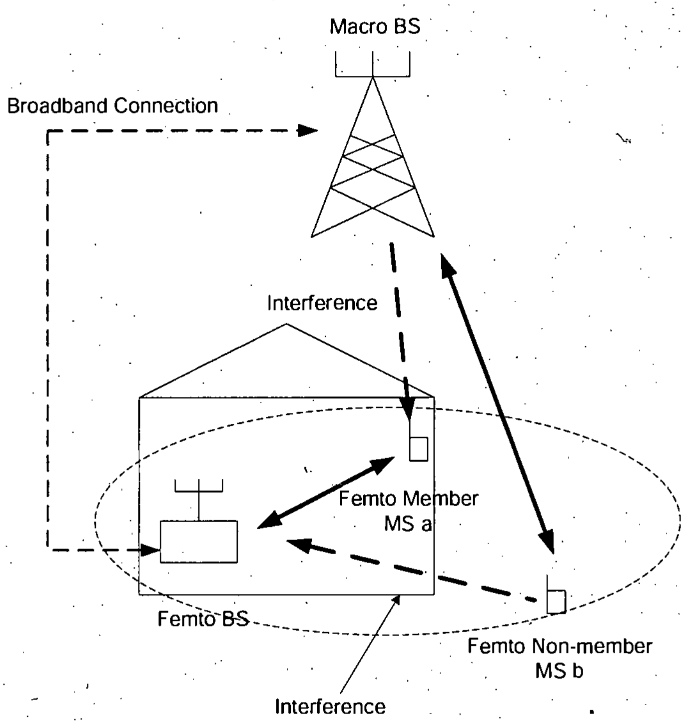


Figure 2.6: *Femtocell*

2.1.3.1 Femtocell

Femtocell is a new concept that allows users to install a small base station typically in indoor environments using licensed spectrum [30] [31] [32]. It connects to wireless networks by a broadband connection such as asymmetric digital subscriber line (ADSL), shown in Figure 2.6. The aim of this technology is to solve the problem of indoor coverage for cellular radio systems. The mobile users can reduce the cost because normally the broadband connection is cheaper than wireless connection. The throughput for indoor environments can also be increased because the whole licensed spectrum can be used by the femtocell base station. From an operator's point of view, femtocell technology can also reduce the cost of building a radio network and network maintenance expense because the macrocell base station only needs to cover those users who can not connect via a femtocell base station and it reduces the number of required macrocell base stations and the throughput as well as coverage requirements of each macrocell.

On the other hand, the owner of a femtocell base station may just want a few users to access his femtocell since he pays the expense of maintenance and the broadband connection. For example, a closed subscriber group (CSG) femtocell base station is defined in 802.16m [33] and it is accessible only to mobile stations which are members of this femtocell except for emergency service. In this case, users which are covered by the femtocell base station but can not access this femtocell will have to suffer strong interference and may not be able to connect with its own macrocell base station/femtocell base station. Therefore, the cognitive radio technologies can be used to deal with the coexistence of femtocell and macrocell and thus mitigate the interference in order to reduce the effect on the macrocell connected users.

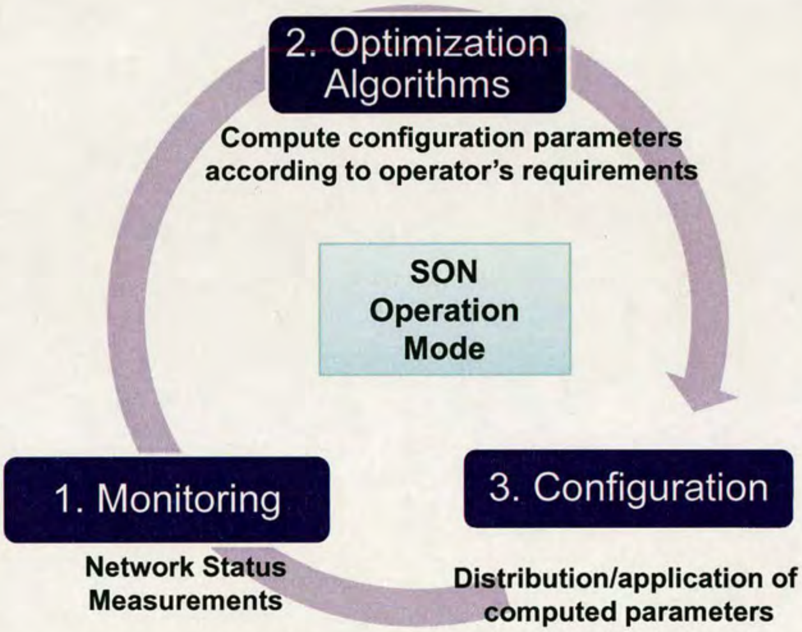


Figure 2.7: *Self-organising network*

2.1.3.2 Self-organising network

Self-organising network (SON) is another hot topic which is included in System Description Document of 802.16m [33]. Current networks are managed manually and it is quite difficult for large-scale systems due to growing complexity. Highly skilled engineers are needed therefore increasing the cost of operators. The aim of SON is try to achieve an automatic network management system which removes human intervention from the "control/management loop" and increases the system efficiency and system reliability. Further more, it can also place the intelligence in the network and simplify overall network management tasks.

Typically, a SON operation has three steps, network status measurements, parameter optimisation, and configuration, shown in Figure 2.7. The network status measurements may include interference measurements, spectrum usage of adjacent cells, etc; and the optimised parameters may be transmission power, modulation and coding scheme, spectrum allocation etc. It is clear that SON system is just like a cognitive radio system and both of them are intelligent and can dynamically adjust according to the surrounding environment. Thus the algorithms used in cognitive radio can be directly used in SON.

2.2 Interference channel

An interference channel model is a scenario where multiple independent transmitters try to communicate their separate information to corresponding receivers using the same frequency [34]. The receiver will not only receive the signals but also receive the interference transmitted by the other transmitters. The study on interference channel is trying to answer two questions:

1. What is the capacity bound of the interference channel?
2. How can the capacity be achieved?

This issue has already been discussed for more than thirty years, and a review of interference channel and many basic results of interference channel can be found in [34] and [35]. In this section, the fundamental results are given to help to understand interference mitigation of cognitive radio. We also assume that the transmitters do not have the side information which includes the transmitted signals of other transmitters.

A simple model of interference channel is presented in Figure 2.8. There are two transmitters and two receivers in this figure (System 1 and System 2). Without loss of generality, we assume that it is a Gaussian interference channel, which means that the two data channels are added white Gaussian noise (AWGN) channels. Then the received signals in both receivers y_1 and y_2

can be expressed as:

$$y_1 = s_1 + \sqrt{a_{21}} s_2 + z_1 \quad (2.1)$$
$$y_2 = \sqrt{a_{12}} s_1 + s_2 + z_2$$

The s_1 and s_2 are transmitted signals with power of P_1 and P_2 respectively. In the receiver, the z_1 and z_2 are added Gaussian noise with zero mean and variances σ_1^2 and σ_2^2 respectively. a_{12} and $a_{21} \geq 0$ are the interference channel power gain which determine the achievable capacity of Gaussian interference channel.

Generally, the interference channel model can be classified into the following classes according to the value of a_{12} and a_{21}:

No interference When $a_{12} = a_{21} = 0$, the interference channel becomes two parallel independent AWGN channels. Therefore, the capacity region for this scenario is a rectangle.

$$R_i \leq C_i = \frac{1}{2}\log(1 + \frac{P_i}{\sigma_i^2}) \quad (2.2)$$

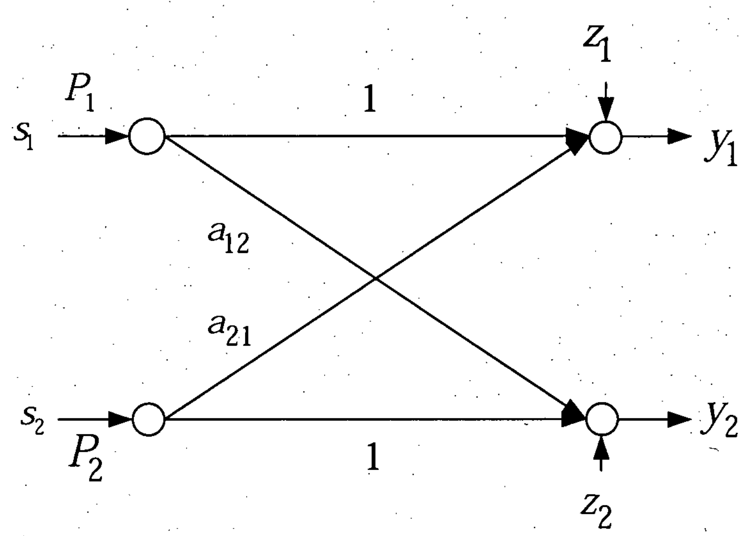


Figure 2.8: *Gaussian interference channel*

Very strong interference When a_{21} and a_{12} satisfy the following conditions simultaneously,

$$a_{21} \geq (P_1 + \sigma_1^2)/\sigma_2^2 \quad (2.3)$$

$$a_{12} \geq (P_2 + \sigma_2^2)/\sigma_1^2 \quad (2.4)$$

it is the called very strong interference case. Then the interference is no harm to data signals and the capacity region is just like no interference scenario [34]. This is because in this case it is possible for both receivers to estimate the interfering signal in a first decoding step, and then subtract it away.

Strong interference When a_{21} and a_{12} satisfy the following conditions simultaneously,

$$a_{21} \geq \sigma_1^2/\sigma_2^2 \quad (2.5)$$

$$a_{12} \geq \sigma_2^2/\sigma_1^2 \quad (2.6)$$

then it is called the strong interference scenario. The capacity region is given as follows [36] [37] [38], shown in Figure 2.9:

$$0 \leq R_1 \leq C_1 \quad (2.7)$$

$$0 \leq R_2 \leq C_2$$

$$0 \leq R_1 + R_2 \leq \min\left[\frac{1}{2}\log(1 + \frac{P_1 + a_{21}P_2}{\sigma_1^2}), \frac{1}{2}\log(1 + \frac{P_2 + a_{12}P_1}{\sigma_2^2})\right]$$

Weak and moderate interference When a_{21} and $a_{12} \in (0, 1)$, it is called the weak or moderate interference scenario. The completed capacity region is unknown. Even when $a_{21} = 0$ or $a_{12} = 0$ and the other coefficient $\in (0, 1)$, the capacity region is still unknown.

2.3 Multiple antennas technologies

Multiple-antenna techniques are one of the most important technological breakthroughs in the radio communication field during the last ten years [39]. It utilises space-time signal processing and multiple antennas to obtain higher capacity or transmission reliability for wireless communication. The basic form of a multiple antenna system, multiple-input multiple-output (MIMO) system, uses multiple antennas at both the base stations and the user terminals. The system

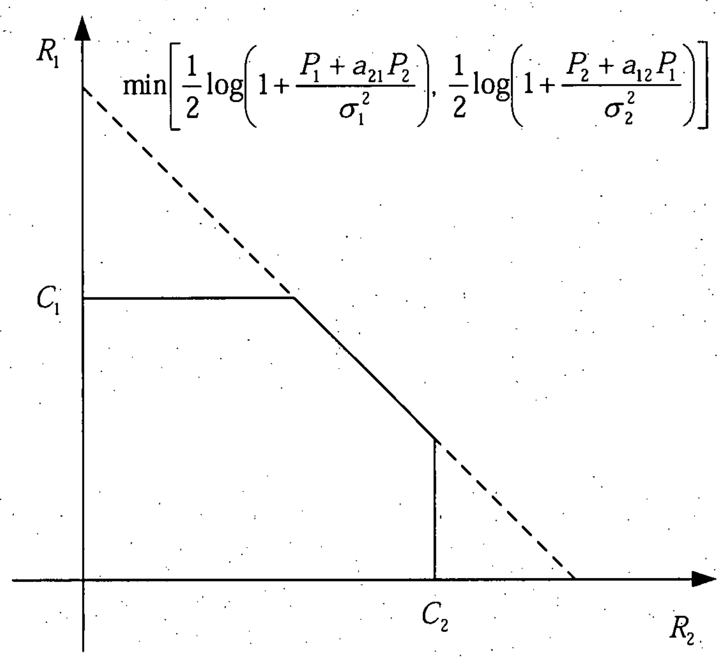


Figure 2.9: *Capacity region for the strong interference scenario*

model of MIMO is shown in Figure 2.10, which is compared with a single input single output (SISO) system. When only one side of the radio system, i.e. the transmitter or receiver, has multiple antennas, the system can be called multiple input single output (MISO) or single input multiple output (SIMO) system. The system models of MISO and SIMO are shown in Figure 2.11. The key feature of multiple-antenna systems is that this technology turns multipath propagation into a benefit for users, which normally is considered as a defect in wireless transmission. Initial work in this area shows that a MIMO system can be seen as multiple parallel SISO systems without any co-channel interference due to the independent of fading between each pair of transmitter and receiver antennas [40]. To summarise the literature about multiple-antenna systems, we can conclude that this technique can offer three kinds of advantages:

1. Multiplexing gain; The MIMO technologies offer a capacity increase without increasing transmitted power and extending bandwidth. This gain, the so called multiplexing gain, is realised though transmitting multiple independent data streams from multiple antennas. The receiver can separate the data if the channel is ideal (the rank of channel matrix is greater than or equal to the number of independent data signals) and the channel state

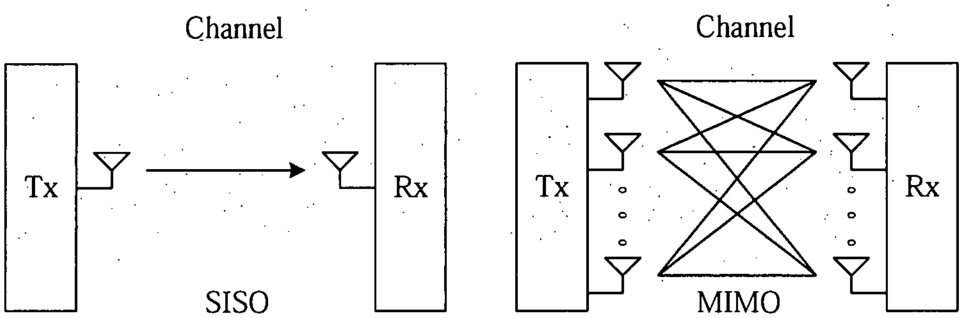


Figure 2.10: *SISO & MIMO system models*

information is known.

2. Space diversity gain; Diversity is a strong technology to reduce the effect of the fading in radio systems. The basic ideal of diversity is to transmit the same data through independent channels. For a MIMO system, the channel between each antenna pair is independent (ideal channel assumption) and therefore it can offer diversity gain. Moreover, the diversity gain can improve the quality of received signals in fading environments, therefore improves the capacity performance and/or reliability.

3. Interference cancellation; Due to the independence of the fading channels, the MIMO system has ability to avoid/mitigate the interference to/from other co-channel systems through selecting suitable transmitted/received vectors.

Overviews about MIMO technologies can be found in [39] [41]. Since our focus is interference mitigation for cognitive radio in this thesis, only related topics in MIMO technology are reviewed, including the basic MIMO capacity, the detection technologies, precoding, smart antennas, and multiple antennas in cognitive radio.

2.3.1 Singular value decomposition

The singular value decomposition (SVD) is the theoretical foundation of MIMO technology. It has been used when MIMO technology was introduced in 1995 [42]. In most of papers,

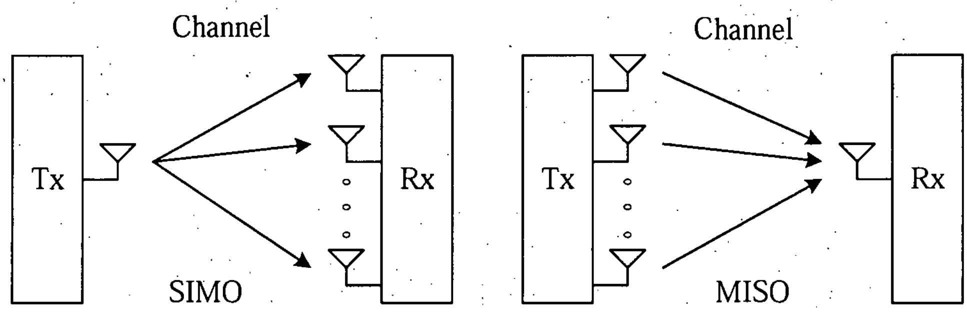


Figure 2.11: *SIMO & MISO system models*

this is used to calculate the capacity of MIMO system and to prove that MIMO system can offer a capacity gain which is proportional to the rank of channel matrix when the channel state information is known. In this method, the detection task is to recover each transmitter's signal at the receiver. Its basic structure is shown in Figure 2.12.

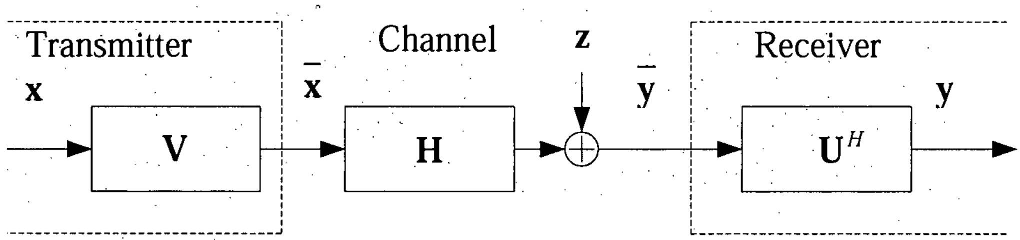


Figure 2.12: *Singular value decomposition block diagram*

Assume that there are N antennas in the transmitter and receiver respectively, and the elements of channel matrix $\mathbf{H}$ are independent. It means that the rank of $\mathbf{H}$ is equal to N. Then the channel information matrix $\mathbf{H}$ can be decomposed as:

$$\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^H \tag{2.8}$$

where $\mathbf{U}$ and $\mathbf{V}$ are both unitary matrices respectively, and $\mathbf{V}^H$ is the conjugate transpose of matrix $\mathbf{V}$. $\mathbf{D} = \text{diag}\ \{\sqrt{\lambda_1}, \sqrt{\lambda_2}, \cdots, \sqrt{\lambda_N}\}$ is a diagonal matrix of the singular values of the channel state matrix $\mathbf{H}$, and λ_i is the eigenvalue of $\mathbf{H}$. The transmitted signals $\mathbf{x}$, whose elements are independent, are precoded by the matrix $\mathbf{V}$ at the transmitter and is expressed as $\overline{\mathbf{x}}$. Therefore

$$\overline{\mathbf{x}} = \mathbf{V}\mathbf{x} \tag{2.9}$$

The received signals $\overline{\mathbf{y}}$ are filtered by $\mathbf{U}^H$ at receiver, as shown in Equation (2.10).

$$\begin{aligned} \mathbf{y} &= \mathbf{U}^H\overline{\mathbf{y}} = \mathbf{U}^H\mathbf{H}\overline{\mathbf{x}} + \mathbf{U}^H\mathbf{z} \\ &= \mathbf{U}^H\mathbf{H}\mathbf{V}\mathbf{x} + \overline{\mathbf{z}} = (\mathbf{U}^H\mathbf{U})\mathbf{D}(\mathbf{V}^H\mathbf{V})\mathbf{x} + \overline{\mathbf{z}} \\ &= \mathbf{D}\mathbf{x} + \overline{\mathbf{z}} \end{aligned} \tag{2.10}$$

where $\mathbf{z}$ is the added Gaussian noise vector with unity power, and $\overline{\mathbf{z}}$ is filtered by $\mathbf{U}$, defined as

$$\overline{\mathbf{z}} = \mathbf{U}^H\mathbf{z}.$$

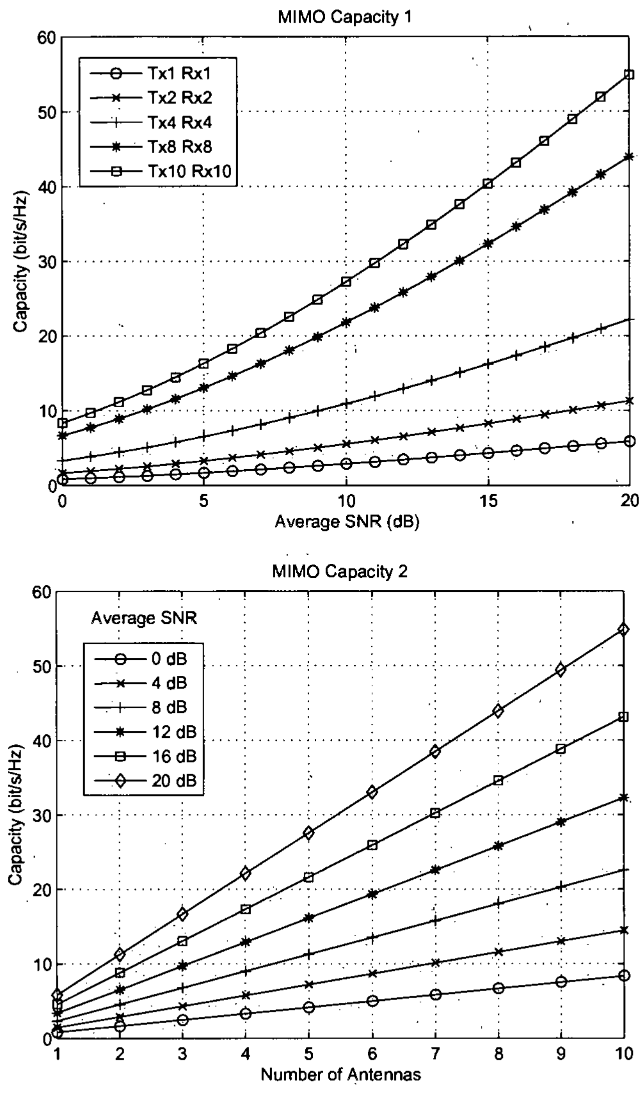


Figure 2.13: *Capacity of MIMO systems*

It is obvious from Equation (2.10) that the MIMO system can be seen as $N = \text{rank}\,(\mathbf{H})$ parallel channels without interference because $\mathbf{D}$ is a diagonal matrix. Since $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, the transmitted power of signals and the received noise power will not be increased by this transformation.

The capacity of the MIMO channel can be expressed as Equation (2.11) when equal transmitted power is applied to each antenna [40]

$$C = \log\det[(\mathbf{I}_{N_R} + (\xi/N_T)\mathbf{H}\mathbf{H}^H] \tag{2.11}$$

where ξ is the average signal to noise power ratio (SNR); N_T, N_R are the number of antennas at the transmitter and receiver, respectively; $\mathbf{I}_{N_R}$ is identity matrix with size N_R; $\det(\cdot)$ is the matrix determinant. The capacity of a MIMO system corresponding to the average SNR and the number of antennas ($N_T = N_R$) are shown in Figure 2.13. It is clear that increasing the number of antennas can greatly increase the capacity of system.

When the water-filling algorithm (optimal power and bit loading algorithm) is used, the maximum value of capacity can be achieved [39]:

$$P = \Sigma_i(\mu - \lambda_i^{-1})^+ \tag{2.12}$$

$$C = \Sigma_i(\log(\mu\lambda_i))^+ \tag{2.13}$$

where P is the total power; λ_i is the eigenvalue of the channel matrix and μ is the so-called water level which satisfies Equation (2.12). $(\cdot)^+$ is the function

$$(x)^+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \tag{2.14}$$

This algorithm cannot be easily implemented in practical systems since the number of transmitted bits must be an integer. Simple approximations from water-filling results are not necessarily optimal. A greedy algorithm, which is the optimal discrete bit loading and power allocation algorithm, is introduced by [43]. Some algorithms with reduced complexity have also been advanced [44].

2.3.2 Linear detection and linear precoding

The optimal detector for MIMO in the sense of minimising error probability is the maximum likelihood (ML) detector. However, this method has to search in the full space to obtain results. Some fast algorithms for sphere decoding have been developed to reduce the complexity [45]. In this subsection our interest is in linear algorithms, and two kinds of linear detector/precoding technologies, including zero-forcing (ZF) and the linear minimum mean squared

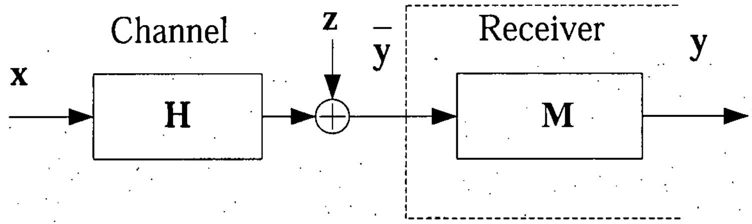


Figure 2.14: *Linear detector block diagram*

error (MMSE), are discussed.

The architecture of the linear detector is presented in Figure 2.14. The received signals are filtered by a transforming matrix $\mathbf{M}$. The ZF detector cancels all the interference from other independent streams, and the MMSE try to achieve the best trade-off between interference and noise. The difference between ZF and MMSE is the transformation matrix. For the ZF, the transformation matrix $\mathbf{M}$ is the left pseudo-inverse of channel matrix $\mathbf{H}$:

$$\mathbf{M} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H \tag{2.15}$$

For MMSE, the decorrelation matrix $\mathbf{M}$ is [41]:

$$\mathbf{M} = (\mathbf{H}^H\mathbf{H} + N_R/\xi\mathbf{I}_{N_T})^{-1}\mathbf{H}^H \tag{2.16}$$

The performance of the MMSE detector tends to that of the ZF detector when the signal to noise power ratio is high. It is well known that ZF will enhance the effect of noise, and, hence, have poor power efficiency.

The linear detector is a receiver side technology. Similar technologies used in transmitter side are precoding which also includes the transmit ZF and transmit MMSE (also named the Wiener filter) [46]. Before signals are transmitted, the data messages are precoded by a transformation matrix, as shown in Figure 2.15. This transformation matrix is the right pseudo-inverse of channel state matrix for the transmit ZF technology (equation (2.17)) [46] [47]. This method will increase the transmission power and decrease the power efficiency, if the matrix is ill-conditioned. For transmit MMSE, the transformation matrix is more complex and can be found

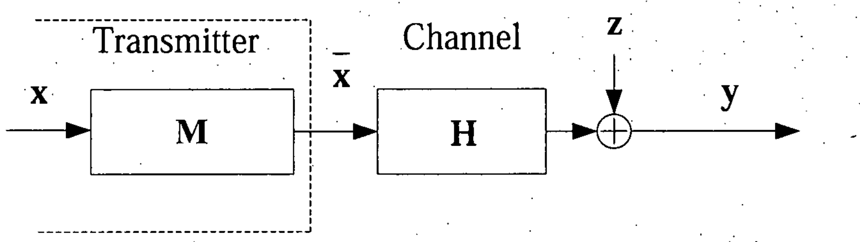


Figure 2.15: *Linear precoding block diagram*

in [46].

$$\mathbf{M} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1} \tag{2.17}$$

2.3.3 Spatial decision feedback equalisation

The disadvantages of linear equalisation and precoding can be overcome by spatial decision feedback equalisation (DFE) [47] [48]. This method is similar to successive interference cancellation (SIC) in multi-user detection [49].

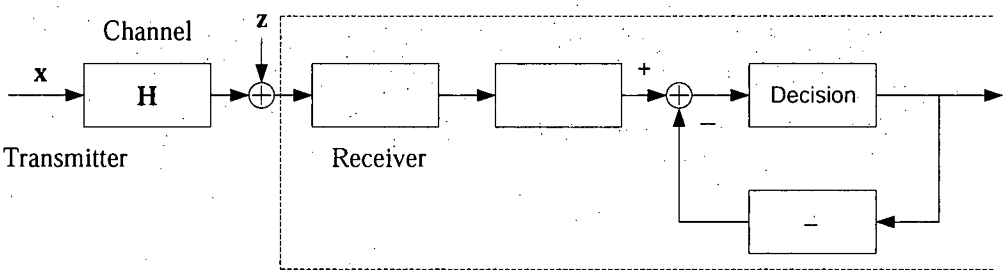


Figure 2.16: *Decision feedback equalisation block diagram*

The block diagram of the DFE is shown in Figure 2.16. The receiver consists of a feed-forward matrix $\mathbf{F}$, a diagonal matrix $\mathbf{G}$, and a feedback matrix $\mathbf{B}$. These three transformation matrices can be obtained from the channel state information matrix $\mathbf{H}$. The channel state information matrix can be decomposed as: (QL decomposition)

$$\mathbf{H} = \mathbf{F}^H\mathbf{S} \tag{2.18}$$

where $\mathbf{F}$ is a unitary matrix; $\mathbf{S}$ is a low triangular matrix; Then define a matrix $\mathbf{S}$ as

$$\mathbf{B} = \mathbf{GS} \tag{2.19}$$

where $\mathbf{G} = \text{diag}\ \{s_{11}^{-1}, s_{22}^{-1}, \cdots, s_{NN}^{-1}\}$, N denotes the minimum of N_R and N_T. The DFE is a receiver-side technology for MIMO detection. In the first stage the antenna which is decoded is only disturbed by noise. The data for the current antenna is decoded depending on the previous stage results. If the SNR of the received signals is low, this detector may lead to inaccurate decoding for the antennas. This is because incorrect data decisions will cause increased interference, an effect called error propagation. If the feedback part is moved to transmitter side, this problem can be overcome. In this case, the channel state matrix $\mathbf{H}$ is decomposed in an LQ sense (a low triangular matrix multiplies a unitary matrix) when the feed-forward matrix is also moved to transmitter side. However, the feedback operation in the transmitter will lead to the increase of transmitter power.

The above decomposition is described for the zero-forcing case. For lower SNR, some performance advantage may be achieved when the transformation matrix is generated using the MMSE criterion.

2.3.4 Non-linear precoding

The disadvantage of the DFE is its performance will greatly degrade due to error propagation when the SNR is low. Although moving the feedback matrix to transmitter can overcome this problem, this will lead to the increase of transmitter power which will limit system performance, and, thus lower power efficiency. To overcome these shortcomings, the Tomlinson-Harashima precoding (THP) technique can be used. This method was initially proposed for mitigating inter-symbol interference of SISO channels [50]. It can also be used in MIMO-DFE systems [47]. The block diagram of THP is shown in Figure 2.17.

The THP in fact is a modulo operation in the feedback loop. It constrains the encoded signals in a square area which includes the signal constellation. For example, as shown in Figure 2.18, if

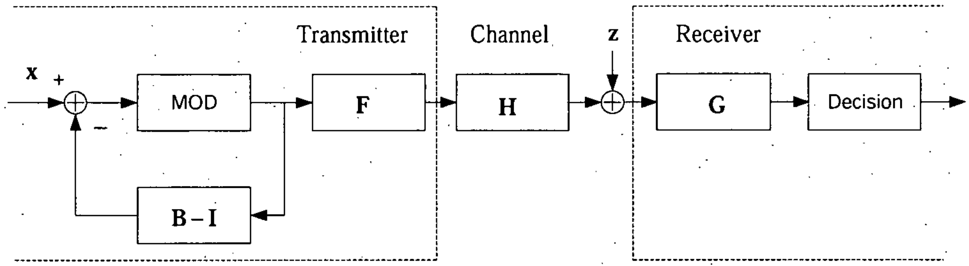


Figure 2.17: *Tomlinson-harashima precoding block diagram*

the constellation (M-QAM modulation) is $a_I + ja_Q$, where $a_I, a_Q \in \{\pm 1, \pm 3, \cdots, \pm(\sqrt{M} - 1)\}$, the constellation is bounded by the square area with width $2\sqrt{M}$. In the receiver, the same modulo operation is utilised before decision model to resume the message.

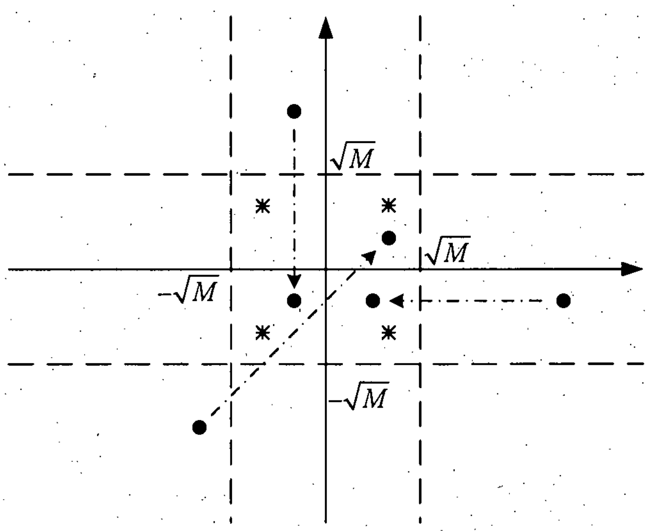


Figure 2.18: *Modulo operation*

After the modulo and feedback operations in the transmitter, for an ideal case, the signals can be assumed to be uniform distributed in the constrained area. This leads to a slight enhancement of transmitted power, but it is much less than if the modulo operation were not used at all.

2.3.5 Smart antennas

The concept of smart antennas was proposed before MIMO technology. The main idea behind smart antenna technology is transmitting or receiving the signals using directional information via signal processing to avoid interference to other co-channel receivers or to maximise the received SNR. The corresponding signal processing is carried out in the multi-antenna transmitters or receivers. The typical technologies for smart antenna are maximum ratio transmission (MRT) and beamforming.

The MRT is a two-side (transmitter and receiver) technique. It transmits a single data stream over multiple antennas after multiplication by a direction vector. In the receiver, the signals are multiplied by a weight vector to recover the signal. The direction vector is the eigenvector corresponding to the maximum eigenvalue of the channel matrix [51]. This is the optimal transmission strategy in the terms of maximising the received SNR or in the terms of mutual information. Maximum ratio combination (MRC) is a special case for MRT in the case of a SIMO system. When multiple data streams are transmitted, an extended MRT is also proposed in [52].

Beamforming is another method to transmit signals for a multiple antenna system. It can be used in the transmitter-side or the receiver-side. The key idea is to transmit signals without interference. For the transmitter-side technique, the transmitted data stream is multiplied by a direction vector, which is orthogonal to the other rows in the channel matrix. This means that the signal is transmitted to one receiver and other receivers do not receive the signals. In this case, the interference is mitigated to zero [52]. The receiver-side beamforming technique is similar with the transmitter-side one. The received signal vector is multiplied by a row vector, which is orthogonal to channel of interfering users, thus the interference from other users is zero.

2.3.6 Comparison of MIMO detector

The aim of this subsection is to compare the performance of different detection and precoding methods for MIMO via simulation. These methods include linear equalisation (ZF detector), DFE, and THP. Simulation conditions are shown in Table 2.1.

The simulation results are shown in Figure 2.19. The nonlinear pre-coding method, THP, achieves the best performance among all the methods because of the usage of modulo operation

Number of transmitter antennas	4
Number of receiver antennas	4
Modulation type	16-quadrature amplitude modulation (16-QAM)
Average power of transmitted signals	0 dB (compared to 1.0)
Channel	Rayleigh fading channel, zero mean, unit power

Table 2.1: *Simulation conditions of MIMO detectors*

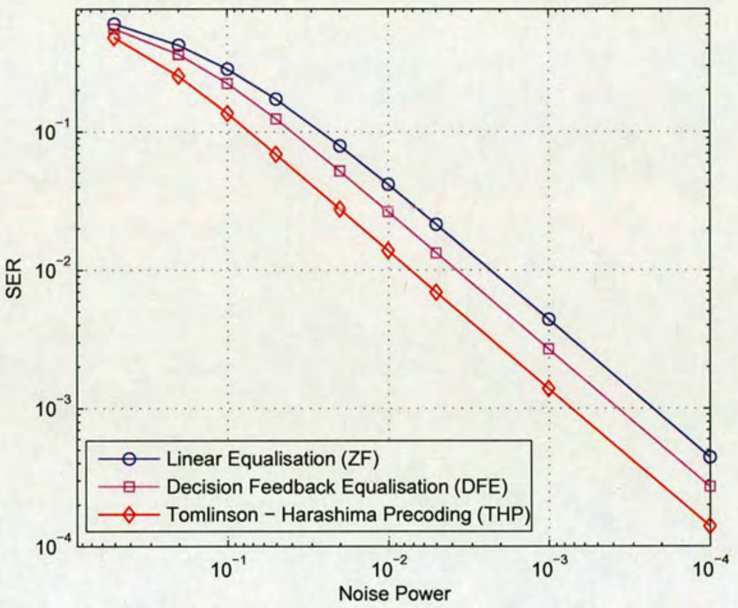


Figure 2.19: *Comparison of different MIMO detectors*

in the transmitter side. The effect of modulo can be seen as virtually increasing the transmission power without really consuming extra power. However high noise may lead to a erroneous modulo operation and thus the worse performance. The linear equalisation method achieves the poorest performance since the decorrelation matrix increases the power of noise. The DFE method does not increase the effect of noise, but error propagation degrades its performance when the SNR is low.

2.4 Conclusion

The cognitive radio is an intelligent radio system which allows multiple radio systems dynamically to share the spectrum resource. It can increase the spectrum efficiency and improve the QoS of users. Typically, a cognitive radio can be classified as an overlay, underlay, or interweave system. Femtocell and SON are two important applications of cognitive radio systems. A major technical challenge of cognitive radio is co-channel interference in the physical layer. When the coexisting radio systems are unsynchronised, the interference becomes more serious.

Multiple antenna is a technology which can supply multiplexing gain and diversity gain. Theoretically, the capacity of a multiple antenna system will linearly increase according to increase of the number of antennas. This technology can be used in cognitive radio systems to deal with the co-channel interference. In this chapter, several precoding and detection algorithms have been presented and compared for a better understanding in the following chapters in which the linear precoding algorithms are used to manage the interference in cognitive radio environments. The single system beamforming methods in which the radio systems precode independently are presented in Chapter 3 and Chapter 4 discusses the joint system beamforming where both the interference channel and data channel information of the primary system and the secondary system are known by the two transmitters.

Chapter 3
Single System Beamforming

As discussed in the previous chapters, one major problem of cognitive radio is co-channel interference (CCI). The single system beamforming methods, in which transmitters of the co-existing radio systems precode independently according to their own channel state information (CSI), are presented to cancel CCI in multiple input single output (MISO) cognitive radio (CR) system in this chapter. There are two key contributions in this chapter: an optimal interference-constrained (IC) precoding method is proposed to maximise the performance of the secondary system while keeping the interference to the primary system below a given limit; and then we analyse and compare the performances of single system beamforming methods in a coexisting radio environment.

Firstly, Section 3.1 gives a brief introduction of the MISO CR system and discusses why the beamforming technologies are needed for a such radio environment; then Section 3.2 describes the system model and basic assumptions used in this chapter. The known and proposed precoding approaches are presented in Section 3.3 with performance analysis. Section 3.4 compares these approaches and gives a summary. Simulation results comparing the performances of these algorithms under various conditions are given in Section 3.5; finally, this chapter is concluded in Section 3.6.

3.1 Introduction

Cognitive radio (CR) has received much attention due to the lack of radio spectrum resources and the low usage statistics of existing spectrum allocations [4] [7]. The key idea of cognitive radio is the coexistence of multiple radio systems in overlapping frequency and/or time slots. Normally these two radio systems are called the primary system and the secondary system (coexisting system) respectively.

Cognitive radio can be classified into two categories according to the interference models. The first one is the initial idea of the cognitive radio, called here "ordinary" cognitive radio, which refers to the coexisting system only utilising those frequency bands or time slots which the primary system temporarily does not use so that the secondary system will not affect the performance of the primary system and be interfered by the primary system. In this concept, the radio resource has only two conditions, reusable and un-reusable, depending on whether co-channel interference exists [53]. However, in real systems, apart from co-channel interference due to the reuse of spectrum, there exists other interference to radio systems, such as environmental noise, thermal noise, and so on. Therefore, the co-channel interference can be seen as a kind of "noise" and it may be controlled. Whether and how the radio resource can be reused depends on the total power of the noise. According to this, the second type of cognitive radio is the extension of the initial concept of cognitive radio, called here "general" cognitive radio, in which all the systems utilise the frequency bands or time slots at same time, whether there is interference or not. Thus, ordinary cognitive radio is a special case of "general" cognitive radio where the co-channel interference does not exist in the secondary user side. In the "general" cognitive radio scenario, the primary system has to suffer co-channel interference from the secondary system therefore degrading its performance. How to eliminate the interference is the key factor in such coexisting radio systems.

Multiple antenna technologies may be a potential solution for mitigating interference in coexisting environments. It is one of the most important technological breakthroughs in recent years. This technology employs multiple antennas in the transmitter side and/or receiver side to achieve significant improvements in system performance, such as capacity, bit error rate (BER), and so on [40] [39]. Generally, multiple antennas can supply multiplexing gain, diversity gain, and co-channel interference suppression to the wireless system because of the independent channel fading between different pairs of antennas.

Consider the downlink of a coexisting environment with multiple antennas. Since complex equipment is usually not used in the terminal side because of cost and power consumption, multiple antennas are often only employed in the base station side. Vector precoding technology is therefore used to improve system performance. Clearly, it is a typical MISO interference channel (IFC). In the case where the two systems know all the transmitted signals, dirty paper coding (DPC) is seen as the optimal approach to maximise the sum capacity performance [54] [55]. However, our major interest in CR system is not the sum capacity since the two systems have different priorities, and the particular challenge of the cognitive radio is that both the transmitters and receivers are distributed and may be unable to coordinate with each other. Thus we consider approaches which do not need knowledge of the other system's transmitted signals. In Larsson and Jorswieck's work [56] [57], they discussed the capacity region bound for MISO IFC based on game theory, but did not specify the vector precoding algorithms for a given performance requirement for the primary system. The maximum ratio transmission (MRT) method in [51] maximises the received signal-to-noise ratio (SNR), but it does not consider the interference to the other radio system and therefore degrades its performance. The zero-forcing (ZF) method, which comes from multiple input and multiple output multi-user detection (MIMO-MUD) techniques [58], perfectly mitigates the interference to other radio systems. However, it may degrade the power of desired signals and lose some of the diversity gain of the channel. The method proposed by Li, Trappe, and Yates in [59] for secret communications in the MISO case maximise the secrecy capacity, which is equal to the difference between message channel capacity and interference channel capacity. In this method the interference power might be small in some cases, but it also might be strong when doing this lead to a significant performance increase for the desired user. However, this is not allowed for the cognitive radio environment since usually the performance of the primary system should be guaranteed and the interference power should be controlled below a certain value.

In this chapter, our focus is on finding linear precoding solutions for the downlink of coexisting environments. We consider the single system case in which independent radio systems only know their own channel state information for the two terminals. Firstly, we present and prove the optimal interference free (IF) approach, which is equivalent to TxZF in MIMO system in [46]; then a novel linear precoding approach, the interference constrained (IC) linear precoding algorithm, is proposed when the primary system can suffer controlled co-channel interference; moreover, we compare these linear approaches under different assumptions of channel state information. The simulation results of these approaches under variable channel environments show that the proposed precoding algorithm can maximise the utilisation of multiple antennas and greatly improve the system performance under reasonable constraints.

3.2 System model

A block diagram of the multiple-input and single-output cognitive radio (MISO-CR) system for downlink transmission is shown in Figure 3.1. Suppose that there are two radio systems, system A and system B. System A is the primary system and system B is the secondary system. It is representative of a typical coexisting environment. The base stations of system A and system B

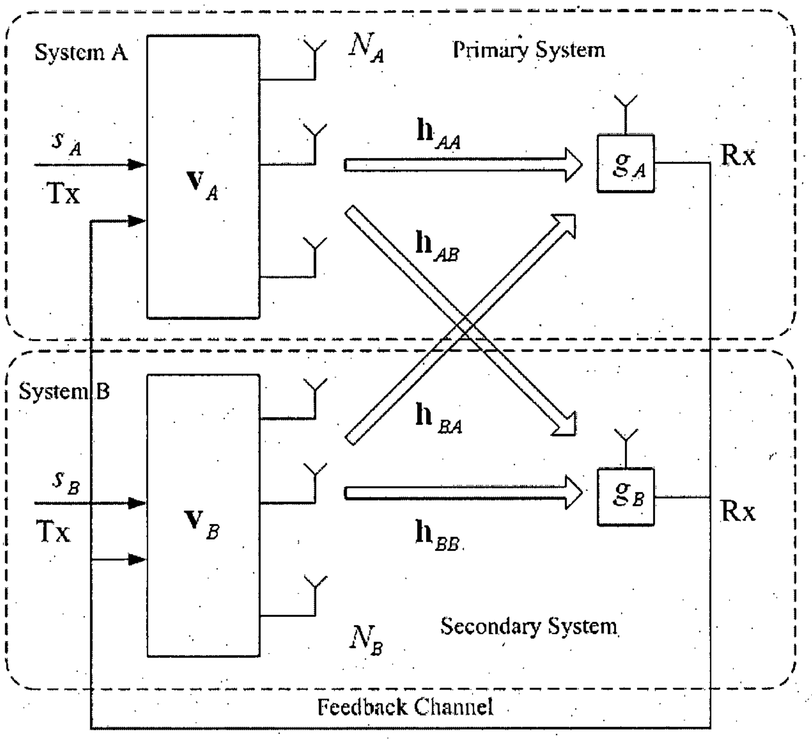


Figure 3.1: *Block diagram of MISO-CR system showing the base stations of system A & B transmitting to users A & B*

have N_A and N_B antennas respectively. We assume that $N_A \geq 2$ and $N_B \geq 2$. In order to avoid the issue of designing space-time precoders in the transmitters, we assume that the messages of system A and system B are scalars, expressed as s_A and $s_B \in \mathbb{C}$, where $\mathbb{C}$ is complex space. They are multiplied by the precoding vectors $\mathbf{v}_A \in \mathbb{C}^{N_A \times 1}$ and $\mathbf{v}_B \in \mathbb{C}^{N_B \times 1}$, then transmitted over frequency non-selective radio channels. In Figure 3.1, the row vectors $\mathbf{h}_{AA} \in \mathbb{C}^{1 \times N_A}$, $\mathbf{h}_{AB} \in \mathbb{C}^{1 \times N_A}$, $\mathbf{h}_{BB} \in \mathbb{C}^{1 \times N_B}$, and $\mathbf{h}_{BA} \in \mathbb{C}^{1 \times N_B}$ are channel vectors, whose elements are defined for different channel models. $\mathbb{C}^{N_1 \times N_2}$ is N_1-by-N_2 dimensional complex space.

The interference model is shown in Figure 3.2. Define the channels of base station A to terminal A and station B to terminal B as data channels with fixed weight 1.0; also define the channels of base station A to terminal B and base station B to terminal A as interference channels with scaling channel weight r_{AB} and r_{BA} respectively (interference factors). The scalars r_{AB} and r_{BA} can allow for interference reduction due to waveform design, frequency overlap, etc. When either r_{AB} or r_{BA} are equal to zero, it is an ordinary cognitive radio scenario; otherwise it is the general cognitive radio case.

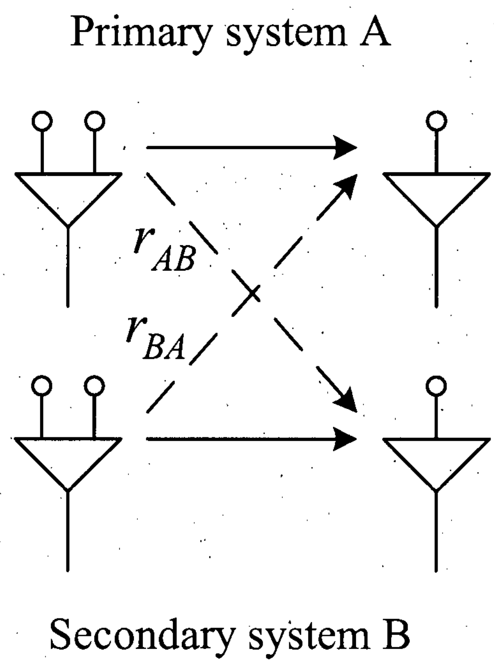


Figure 3.2: *Interference model*

The received signals y_A and y_B are:

$$
\begin{aligned}
y_A &= \mathbf{h}_{AA}\mathbf{v}_A s_A + r_{BA}\mathbf{h}_{BA}\mathbf{v}_B s_B + z_A && (3.1)\\
y_B &= \underbrace{\mathbf{h}_{BB}\mathbf{v}_B s_B}_{\mathrm{I}} + \underbrace{r_{AB}\mathbf{h}_{AB}\mathbf{v}_A s_A}_{\mathrm{II}} + \underbrace{z_B}_{\mathrm{III}} && (3.2)
\end{aligned}
$$

where part I is the desired signal, part II is the co-channel interference (CCI), and part III is the additive noise. Moreover, define $\mathbf{h}_{AA}$, $\mathbf{h}_{BB}$ as message channels (data channels), and $\mathbf{h}_{AB}$, $\mathbf{h}_{BA}$ as interference channels with scaling channel weight r_{AB} and r_{BA} respectively. The noise terms z_A and z_B are independent, identically distributed (i.i.d.) complex Gaussian random variables with zero mean. Their covariances are σ_A^2 and σ_B^2 respectively.

In the receivers, the received signals are multiplied by the complex scalars g_A and g_B respectively. Then a slicer is applied to demodulate the transmitted data. The receivers also estimate their corresponding data channel and interference channel information, and according to different applications, feed back this channel information to the transmitters, as shown in Figure 3.1.

The transmitters will use this information as they compute the beamforming vectors.

We assume that the total transmit powers for both radio system are constrained by

$$\begin{aligned} \mathrm{E}\left[\|\mathbf{v}_A s_A\|^2\right] &= \mathrm{E}\left[|s_A|^2\right] \mathrm{Tr}\left(\mathbf{v}_A \mathbf{v}_A^H\right) \leq P_A \\ \mathrm{E}\left[\|\mathbf{v}_B s_B\|^2\right] &= \mathrm{E}\left[|s_B|^2\right] \mathrm{Tr}\left(\mathbf{v}_B \mathbf{v}_B^H\right) \leq P_B. \end{aligned}$$

where P_A and P_B are the maximum transmission power for the primary system and the secondary system respectively. $\mathrm{E}[\cdot]$ is the mathematical expectation; $\mathrm{Tr}(\cdot)$ is the trace of matrix; $\|\mathbf{h}\|$ is the norm or length of vector $\mathbf{h}$, defined as $(\sum h_i h_i^*)^{1/2}$; $|x|$ is the norm of scalar x, defined as $(xx^*)^{1/2}$.

Without loss of generality, define

$$\mathrm{E}\left[|s_A|^2\right] = P_A \text{ and } \mathrm{E}\left[|s_B|^2\right] = P_B \tag{3.3}$$

then

$$\mathrm{Tr}(\mathbf{v}_A \mathbf{v}_A^H) \leq 1 \text{ and } \mathrm{Tr}(\mathbf{v}_B \mathbf{v}_B^H) \leq 1. \tag{3.4}$$

The received signal-to-interference-noise ratio (SINR) for system A and system B are

$$\mathrm{SINR}_A = \frac{P_A \mathbf{v}_A^H \mathbf{h}_{AA}^H \mathbf{h}_{AA} \mathbf{v}_A}{r_{BA}^2 P_B \mathbf{v}_B^H \mathbf{h}_{BA}^H \mathbf{h}_{BA} \mathbf{v}_B + \sigma_A^2} \tag{3.5}$$

and

$$\mathrm{SINR}_B = \frac{P_B \mathbf{v}_B^H \mathbf{h}_{BB}^H \mathbf{h}_{BB} \mathbf{v}_B}{r_{AB}^2 P_A \mathbf{v}_A^H \mathbf{h}_{AB}^H \mathbf{h}_{AB} \mathbf{v}_A + \sigma_B^2}. \tag{3.6}$$

The normalised mean squared error for system A and system B are defined as:

$$\mathrm{MSE}_A^{nr} = \frac{\mathrm{E}\left[|s_A - g_A y_A|^2\right]}{P_A} \tag{3.7}$$

and

$$\mathrm{MSE}_B^{nr} = \frac{\mathrm{E}\left[|s_B - g_B y_B|^2\right]}{P_B}. \tag{3.8}$$

For fixed precoding vectors $\mathbf{v}_A$ and $\mathbf{v}_B$, the minimum normalised mean squared error (MMSE) for system A and B are

$$\text{MMSE}_A^{nr} = \frac{r_{BA}^2 P_B \mathbf{v}_B^H \mathbf{h}_{BA}^H \mathbf{h}_{BA} \mathbf{v}_B + \sigma_A^2}{P_A \mathbf{v}_A^H \mathbf{h}_{AA}^H \mathbf{h}_{AA} \mathbf{v}_A + r_{BA}^2 P_B \mathbf{v}_B^H \mathbf{h}_{BA}^H \mathbf{h}_{BA} \mathbf{v}_B + \sigma_A^2} \tag{3.9}$$

and

$$\text{MMSE}_B^{nr} = \frac{r_{AB}^2 P_A \mathbf{v}_A^H \mathbf{h}_{AB}^H \mathbf{h}_{AB} \mathbf{v}_A + \sigma_B^2}{P_B \mathbf{v}_B^H \mathbf{h}_{BB}^H \mathbf{h}_{BB} \mathbf{v}_B + r_{AB}^2 P_A \mathbf{v}_A^H \mathbf{h}_{AB}^H \mathbf{h}_{AB} \mathbf{v}_A + \sigma_B^2} \tag{3.10}$$

where

$$\widehat{g}_A = \frac{\sqrt{P_A} \mathbf{v}_A^H \mathbf{h}_{AA}^H}{P_A \mathbf{v}_A^H \mathbf{h}_{AA}^H \mathbf{h}_{AA} \mathbf{v}_A + r_{BA}^2 P_B \mathbf{v}_B^H \mathbf{h}_{BA}^H \mathbf{h}_{BA} \mathbf{v}_B + \sigma_A^2} \tag{3.11}$$

and

$$\widehat{g}_B = \frac{\sqrt{P_B} \mathbf{v}_B^H \mathbf{h}_{BB}^H}{P_B \mathbf{v}_B^H \mathbf{h}_{BB}^H \mathbf{h}_{BB} \mathbf{v}_B + r_{AB}^2 P_A \mathbf{v}_A^H \mathbf{h}_{AB}^H \mathbf{h}_{AB} \mathbf{v}_A + \sigma_B^2}. \tag{3.12}$$

3.3 Single system beamforming

The single system algorithms are used when the two systems cannot exchange their channel state information. Each system independently precodes depending on their own channel state information for their users. Sometimes, one radio system even is not aware of the existence of the other radio system. In this section, we consider precoding methods for system B (the secondary system) and fix the precoding vector for system A (the primary system). Firstly we introduce the known precoding approaches, maximum ratio transmission (MRT) and zero-forcing (ZF). Then the optimal interference free (IF) and optimal interference constrained (IC) techniques are presented and their properties are proved. Each approach is discussed not only considering the performance of system B, but also considering the effect to the primary system (system A). In these approaches, we usually assume that the base station of system B knows the channel state information of the interference channel and/or its own data channel through feedback from the terminals. Although these linear precoding methods are based on single user CR system, we can expend them to multiple user systems using other multiplexing techniques, such as CDMA, FDMA (frequency division multiple access), or TDMA (time division multiple access).

3.3.1 Maximum ratio transmission (MRT)

Typically, there are two kinds of known precoding approaches for MISO systems. One is maximum ratio transmission (MRT) or TxMF [51] [46]. This method maximises the received desired signal power subject to a transmitter power constraint. For system B, the problem is described as follows:

$$\mathbf{v}_B^{MRT} = \arg\max_{\{\mathbf{v}_B\}} |\mathbf{h}_{BB}\mathbf{v}_B|^2 \quad \text{s.t.:} \quad \mathrm{Tr}(\mathbf{v}_B\mathbf{v}_B^H) \leq 1.$$

The solution of this approach is proportional to the conjugate transpose of the normalised message channel:

$$\mathbf{v}_B^{MRT} = \frac{\mathbf{h}_{BB}^H}{\|\mathbf{h}_{BB}\|}. \tag{3.13}$$

This method provides the best performance for system B, but without considering the interference to system A. According to equation (3.6) and (3.10) , it achieves the maximum SNR and minimum MSE for user B [51] [46] for a given precoding vector for the primary system. The maximum diversity gain against the fading channel is also obtained. In addition, only the message channel information for system B is needed. The received power for the secondary system when MRT is used, G_B^{MRT}, can be obtained form (3.1), (3.3) and (3.13):

$$G_B^{MRT} = \mathrm{E}\left[|\mathbf{h}_{BB}\mathbf{v}_B^{MRT}s_B|^2\right] = P_B\|\mathbf{h}_{BB}\|^2$$

The main drawback of MRT in coexisting environments is that it may greatly degrade the performance of system A due to the interference coming from system B. According (3.1), (3.3) and (3.13), the interference generated by system B, I_B^{MRT}, can be written as:

$$\begin{aligned} I_B^{MRT} &= \mathrm{E}\left[|r_{BA}\mathbf{h}_{BA}\mathbf{v}_B^{MRT}s_B|^2\right] = r_{BA}^2 P_B\,|\mathbf{h}_{BA}\mathbf{v}_B^{MRT}|^2 \\ &= r_{BA}^2 P_B \frac{|\mathbf{h}_{BA}\mathbf{h}_{BB}^H|^2}{\|\mathbf{h}_{BB}\|^2} \\ &= r_{BA}^2 \rho_B^2 \|\mathbf{h}_{BA}\|^2 P_B \end{aligned}$$

where ρ_B is the cross-correlation coefficient of the secondary system, defined as:

$$\rho_B = \frac{|\mathbf{h}_{BA}\mathbf{h}_{BB}^H|}{\|\mathbf{h}_{BA}\|\|\mathbf{h}_{BB}\|}. \tag{3.14}$$

3.3.2 Zero-forcing (ZF)

The next method, named zero-forcing (ZF) [58], can perfectly cancel the interference to system A. The key idea of this method is to find a vector which is orthogonal to the interference channel, $\mathbf{h}_{BA}\mathbf{v}_B = 0$. Then, this problem can be expressed as:

$$\mathbf{v}_B^{ZF} = \text{Any } \mathbf{v}_B, \quad \mathbf{v}_B \in \{\mathbf{v} | \, |\mathbf{h}_{BA}\mathbf{v}|^2 = 0 \text{ and } \mathrm{Tr}(\mathbf{v}\mathbf{v}^H) \leq 1\}. \tag{3.15}$$

Define the system B interference self-correlation matrix

$$\mathbf{F}_B = \mathbf{h}_{BA}^H \mathbf{h}_{BA},$$

and hence $\mathrm{rank}(\mathbf{F}_B) = 1$. Thus, it has one non-zero eigenvalue and $N_B - 1$ zero eigenvalues,

$$\lambda(\mathbf{F}_B) = \{\lambda_{non-zero}, 0, \cdots, 0\},$$

where

$$\lambda_{non-zero} = ||\mathbf{h}_{BA}||^2.$$

The eigenvectors corresponding to the zero eigenvalues of the interference self-correlation matrix form the possible solutions of the ZF method. The data channel information for system B is not needed for this approach. A direct solution of ZF is:

$$\mathbf{v}_B^{ZF} = \frac{\overline{\mathbf{v}}_B}{||\overline{\mathbf{v}}_B||} \tag{3.16}$$

and

$$\overline{\mathbf{v}}_B = \mathbf{v} - \frac{\mathbf{h}_{BA}\mathbf{v}}{||\mathbf{h}_{BA}||^2}\mathbf{h}_{BA}^H \quad \mathbf{v} \in \mathbb{C}^{N_B \times 1}, ||\overline{\mathbf{v}}_B|| \neq 0. \tag{3.17}$$

Proof: From (3.16) it is easy to see that if $||\overline{\mathbf{v}}_B|| \neq 0$

$$\mathrm{Tr}(\mathbf{v}_B^{ZF}(\mathbf{v}_B^{ZF})^H) = \frac{\overline{\mathbf{v}}_B^H \overline{\mathbf{v}}_B}{||\overline{\mathbf{v}}_B||^2} = 1$$

and for arbitrary $\mathbf{v}$ which satisfies $||\overline{\mathbf{v}}_B|| \neq 0$, we have

$$\begin{aligned}
\mathbf{h}_{BA}\mathbf{v}_B^{ZF} &= (\mathbf{h}_{BA}\mathbf{v} - \mathbf{h}_{BA}\frac{\mathbf{h}_{BA}\mathbf{v}}{||\mathbf{h}_{BA}||^2}\mathbf{h}_{BA}^H)/||\overline{\mathbf{v}}_B|| \\
&= (\mathbf{h}_{BA}\mathbf{v} - \frac{\mathbf{h}_{BA}\mathbf{h}_{BA}^H}{||\mathbf{h}_{BA}||^2}\mathbf{h}_{BA}\mathbf{v})/||\overline{\mathbf{v}}_B|| = 0.
\end{aligned}$$

Then $\mathbf{v}_B^{ZF}$ is one possible ZF solution.

If there are more than two antennas in base station B, the solution to the ZF algorithm is not unique because there are more than one independent eigenvector corresponding to a zero eigenvalue. An arbitrary choice of the ZF vector will lead to the loss of some diversity gain due to not exploiting the data channel information of user B. How to choose the optimum vector for the MISO-CR case will be discussed in the next subsection.

3.3.3 Optimal interference-free (IF)

The optimal interference-free precoding vector optimises performance of the secondary system whilst avoiding interference to the primary system. It will provide better performance than the ZF method when there are more than two transmit antennas in the base station. We select minimising MSE as our performance criterion. The problem is described as follows:

$$\mathbf{v}_B^{IF} = \arg\min_{\{\mathbf{v}_B\}} \mathrm{MSE}_B^{nr}(\mathbf{v}_B)$$

$$\text{s.t.: } |\mathbf{h}_{BA}\mathbf{v}_B|^2 = 0 \text{ and } \mathrm{Tr}(\mathbf{v}_B\mathbf{v}_B^H) \leq 1$$

Fix the precoding vector $\mathbf{v}_A$, We can see from equation (3.10) that the interference power from the primary system is fixed. Then if we maximise the scalar $||\mathbf{h}_{BB}\mathbf{v}_B||^2$ with the above constraints, the minimising MSE can be obtained.

Theorem 3.1. *Let $\mathbf{h}_{BB}$ and $\mathbf{h}_{BA} \in \mathbb{C}^{1\times N_B}$, $N_B \geq 2$, be the message channel and the interference channel row vectors respectively. If they are not linearly dependent, i.e. $|\mathbf{h}_{BA}\mathbf{h}_{BB}^H| \neq ||\mathbf{h}_{BA}||\,||\mathbf{h}_{BB}||$, the optimal precoding vector $\mathbf{v}_B$ maximising the $|\mathbf{h}_{BB}\mathbf{v}_B|^2$ with constraints $|\mathbf{h}_{BA}\mathbf{v}_B|^2 = 0$ and $\mathbf{v}_B^H\mathbf{v}_B = 1$ is:*

$$\mathbf{v}_B^{IF} = \left[\mathbf{I}_{N_B} - \frac{\mathbf{h}_{BA}^H \mathbf{h}_{BA}}{\|\mathbf{h}_{BA}\|^2}\right] \frac{\mathbf{h}_{BB}^H}{\|\mathbf{h}_{BB}\| \sqrt{(1-\rho_B^2)}} \tag{3.18}$$

and

$$|\mathbf{h}_{BB}\mathbf{v}_B^{IF}|_{\max}^2 = \|\mathbf{h}_{BB}\|^2 (1-\rho_B^2). \tag{3.19}$$

Proof. Let $\mathbf{F}_B = \mathbf{h}_{BA}^H \mathbf{h}_{BA}$ be the interference channel self-correlation matrix, and $\mathbf{G}_B = \mathbf{h}_{BB}^H \mathbf{h}_{BB}$ be the message channel self-correlation matrix. Since they are Hermitian matrices, they can be decomposed as:

$$\mathbf{F}_B = \mathbf{U}^H diag\{\lambda_{BA}, 0, \cdots, 0\}\mathbf{U}$$

and

$$\mathbf{G}_B = \mathbf{M}^H diag\{\lambda_{BB}, 0, \cdots, 0\}\mathbf{M}$$

where $\mathbf{U}$ and $\mathbf{M}$ are unitary matrices [60]. Define

$$\mathbf{U} = [\mathbf{u}_0, \mathbf{u}_1, \cdots, \mathbf{u}_{N_B-1}],$$

$$\mathbf{P} = [\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_{N_B-1}],$$

and

$$\mathbf{M} = [\mathbf{m}_0, \mathbf{m}_1, \cdots, \mathbf{m}_{N_B-1}],$$

where $\mathbf{u}_0$ and $\mathbf{m}_0$ are the eigenvectors corresponding the non-zero eigenvalues of $\mathbf{F}_B$ and $\mathbf{G}_B$ respectively. Without losing generality, define $\mathbf{u}_0 = \mathbf{h}_{BA}^H/\|\mathbf{h}_{BA}\|$ and $\mathbf{m}_0 = \mathbf{h}_{BB}^H/\|\mathbf{h}_{BB}\|$. The vector $\mathbf{m}_0$ is the precoding vector of MRT. Then $\lambda_{BA} = \|\mathbf{h}_{BA}\|^2$ and $\lambda_{BB} = \|\mathbf{h}_{BB}\|^2$. It is clear that

$$\mathbf{U}\mathbf{U}^H = [\mathbf{u}_0, \mathbf{P}][\mathbf{u}_0, \mathbf{P}]^H = \mathbf{u}_0\mathbf{u}_0^H + \mathbf{P}\mathbf{P}^H = \mathbf{I}.$$

The $N_B - 1$ vectors $\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_{N_B-1}$ are independent, unit norm, and orthogonal to $\mathbf{h}_{BA}$ as they correspond to the zero eigenvalues of $\mathbf{F}_B$, therefore they are the basis vectors of the null space of $\mathbf{h}_{BA}$. So if and only if the vector $\mathbf{v}_B$ can be represented as a complex linear combination of these vectors, $|\mathbf{h}_{BA}\mathbf{v}_B|^2 = 0$. Define $\mathbf{v}_B = \mathbf{P}\mathbf{c}$ and $\mathbf{c} = [c_1, c_2, \cdots, c_{N_B-1}]^T$, then

$$\mathrm{Tr}(\mathbf{v}_{\mathrm{B}}\mathbf{v}_{\mathrm{B}}^{\mathrm{H}}) = (\mathbf{P}\mathbf{c})^{\mathrm{H}}\mathbf{P}\mathbf{c} = \mathbf{c}^{\mathrm{H}}(\mathbf{P}^{\mathrm{H}}\mathbf{P})\mathbf{c} = \|\mathbf{c}\|^2 = 1.$$

Therefore we have

$$\begin{aligned}
|\mathbf{h}_{BB}\mathbf{v}_B|^2 &= \mathbf{v}_B^H(\mathbf{h}_{BB}^H\mathbf{h}_{BB})\mathbf{v}_B = \mathbf{v}_B^H\mathbf{G}_B\mathbf{v}_B \\
&= \mathbf{v}_B^H\mathbf{M}^H diag\{\lambda_{BB}, 0, \cdots, 0\}\mathbf{M}\mathbf{v}_B \\
&= \lambda_{BB}\left|\mathbf{v}_B^H\mathbf{m}_0\right|^2 = \lambda_{BB}\left|(\mathbf{P}\mathbf{c})^H\mathbf{m}_0\right|^2 \\
&= \lambda_{BB}\left|\mathbf{m}_0^H\mathbf{P}\mathbf{c}\right|^2 \\
&\le \lambda_{BB}\left\|\mathbf{m}_0^H\mathbf{P}\right\|^2\|\mathbf{c}\|^2 = \lambda_{BB}\left\|\mathbf{m}_0^H\mathbf{P}\right\|^2.
\end{aligned}$$

When

$$\mathbf{c} = \frac{(\mathbf{m}_0^H\mathbf{P})^H}{\|\mathbf{m}_0^H\mathbf{P}\|} = \frac{\mathbf{P}^H\mathbf{m}_0}{\|\mathbf{m}_0^H\mathbf{P}\|},$$

the equality is valid. Then

$$\mathbf{v}_B^{IF} = \frac{\mathbf{P}\mathbf{P}^H\mathbf{m}_0}{\|\mathbf{m}_0^H\mathbf{P}\|} = \frac{(\mathbf{I}_{N_B} - \mathbf{u}_0\mathbf{u}_0^H)\mathbf{m}_0}{\|\mathbf{m}_0^H\mathbf{P}\|}$$

and

$$\begin{aligned}
\left\|\mathbf{m}_0^H\mathbf{P}\right\|^2 &= \mathrm{Tr}(\mathbf{m}_0^H\mathbf{P}\mathbf{P}^H\mathbf{m}_0) \\
&= \|\mathbf{m}_0\|^2 - \left|\mathbf{m}_0^H\mathbf{u}_0\right|^2 = 1 - \rho_B^2
\end{aligned}$$

where

$$\rho_B = \frac{\left|\mathbf{h}_{BA}\mathbf{h}_{BB}^H\right|}{\|\mathbf{h}_{BA}\|\,\|\mathbf{h}_{BB}\|}.$$

So

$$\left|\mathbf{h}_{BB}\mathbf{v}_B^{IF}\right|_{\max}^2 = \|\mathbf{h}_{BB}\|^2(1-\rho_B^2)$$

and

$$\begin{aligned}
\mathbf{v}_B^{IF} &= \frac{(\mathbf{I}_{N_B} - \mathbf{u}_0\mathbf{u}_0^H)\mathbf{m}_0}{\sqrt{(1-\rho_B^2)}} \\
&= \left[\mathbf{I}_{N_B} - \frac{\mathbf{h}_{BA}^H\mathbf{h}_{BA}}{\|\mathbf{h}_{BA}\|^2}\right]\frac{\mathbf{h}_{BB}^H}{\|\mathbf{h}_{BB}\|\sqrt{(1-\rho_B^2)}}.
\end{aligned}$$

□

In fact, the IF method is equivalent to TxZF method which has been discussed in point-to-point MIMO systems in [46]. Here, we adapt it to the MISO-CR scenario.

The optimal precoding vector in (3.18) can be seen to be based on the MRT solution, which achieves the maximum value of received desired signal power, but subtracting out the vector component which causes the interference, while maintaining the unit power constraint. This also shows that in our case the optimal combination of coefficients for the null space of $\mathbf{h}_{BA}$ is linearly proportional to the corresponding coefficients of the MRT solution which is the optimal combination of the signal space.

Moreover, the desired signal power, as shown in (3.19), is not influenced by the power of the interference channel but only depends on the cross-correlation coefficient between the channel vectors, which depends on the direction of the interference channel. The desired signal power is inversely proportional to the square of the cross-correlation coefficient, which is related to the angle between the message channel vector and the interference channel vector, as shown in Figure 3.3. When the two channels are orthogonal, such as $\mathbf{h}_{\mathbf{BA},3}$ and $\mathbf{h}_{\mathbf{BB}}$, this approach achieves as good performance as MRT, which optimises performance of the secondary system when ignoring effect on the primary system. As the correlation coefficient tends to 1.0, the desired power decreases to 0. From (3.10) and (3.19), the normalised MMSE for system B is:

$$\mathrm{MMSE}_B^{nr} = \frac{r_{AB}^2 P_A \mathbf{v}_A^H \mathbf{h}_{AB}^H \mathbf{h}_{AB} \mathbf{v}_A + \sigma_B^2}{P_B \left(1-\rho_B^2\right) \left\|\mathbf{h}_{BB}\right\|^2 + r_{AB}^2 P_A \mathbf{v}_A^H \mathbf{h}_{AB}^H \mathbf{h}_{AB} \mathbf{v}_A + \sigma_B^2}. \tag{3.20}$$

3.3.4 Optimal interference-constrained (IC)

The optimal interference free precoding approach has been discussed in the previous subsection. However, sometimes the primary system may be able to tolerate controlled levels of interference in practice. For example, the received SNR of the primary system is much better than the required SNR for acceptable performance due to good channel conditions. Or we can increase the transmission power of the primary system to obtain better SNR. In this scenario, this headroom may be used to help the secondary system achieve better performance. Keeping this in mind, an interference constrained precoding algorithm is proposed. The problem is described as follows:

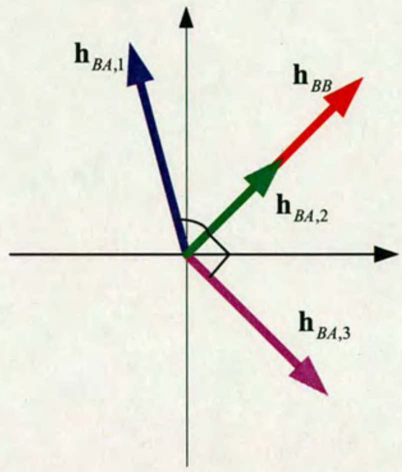


Figure 3.3: *Effect of the angle between interference channel and data channel*

$$\mathbf{v}_B^{IC} = \underset{\{\mathbf{v}_B\}}{\arg\min}\, \mathrm{MSE}_B^{nr}(\mathbf{v}_B)$$

$$\text{s.t.:}\;\; r_{BA}^2 P_B |\mathbf{h}_{BA}\mathbf{v}_B|^2 \leq J \;\text{ and }\; \mathrm{Tr}(\mathbf{v}_B\mathbf{v}_B^H) \leq 1$$

where J is the maximum acceptable interference of the primary system.

The only difference to the IF scheme in the previous subsection is the interference constraint, therefore this problem is also equal to maximising the scalar $|\mathbf{h}_{BB}\mathbf{v}_B|^2$ with the above constraints according to equation (3.10).

Theorem 3.2. *Let $\mathbf{h}_{BB}$ and $\mathbf{h}_{BA} \in \mathbb{C}^{1\times N_B}$, $N_B \geq 2$, be the message channel and the interference channel row vectors respectively. If they are not linearly dependent, i.e. $|\mathbf{h}_{BA}\mathbf{h}_{BB}^H| \neq \|\mathbf{h}_{BA}\|\,\|\mathbf{h}_{BB}\|$, the optimal precoding vector $\mathbf{v}_B$ maximising the $|\mathbf{h}_{BB}\mathbf{v}_B|^2$ with constraints $r_{BA}^2 P_B|\mathbf{h}_{BA}\mathbf{v}_B|^2 \leq J$ and $\mathbf{v}_B^H\mathbf{v}_B = 1$ is:*

$$\mathbf{v}_B^{IC} = \sqrt{1-\beta^2}\mathbf{v}_B^{IF} + \beta\frac{\mathbf{h}_{BB}\mathbf{h}_{BA}^H}{\rho_B}\mathbf{h}_{BA}^H \tag{3.21}$$

where

$$\beta \in \mathbb{R} \;\text{ and }\; 0 \leq \beta \leq \rho_B$$

If

$$\sqrt{\frac{J}{r_{BA}^2 P_B \|\mathbf{h}_{BA}\|^2}} \geq \rho_B \tag{3.22}$$

then

$$|\mathbf{h}_{BB}\mathbf{v}_B^{IC}|^2_{\max} = \|\mathbf{h}_{BB}\|^2 \text{ and } \beta = \rho_B \tag{3.23}$$

else

$$|\mathbf{h}_{BB}\mathbf{v}_B^{IC}|^2_{\max} = \|\mathbf{h}_{BB}\|^2 |\sqrt{(1-\beta^2)(1-\rho_B^2)} + \beta\rho_B|^2 \tag{3.24}$$

and

$$\beta = \sqrt{\frac{J}{r_{BA}^2 P_B \|\mathbf{h}_{BA}\|^2}} \tag{3.25}$$

Proof. It is clear that the optimal precoding vector $\mathbf{v}_B$ can be written two parts, the no interference component proportional to the optimal IF precoding vector $\mathbf{v}_B^{IF}$, and an interference causing component proportional to the vector $\mathbf{u}_0$, shown in (3.26):

$$\mathbf{v}_B^{IC} = \bar{\alpha}\mathbf{v}_B^{IF} + \bar{\beta}\mathbf{u}_0 \tag{3.26}$$

Let us examine the two constraints:

$$\mathbf{v}_B^H \mathbf{v}_B = (\bar{\alpha}\mathbf{v}_B^{IF} + \bar{\beta}\mathbf{u}_0)^H(\bar{\alpha}\mathbf{v}_B^{IF} + \bar{\beta}\mathbf{u}_0) = |\bar{\alpha}|^2 + |\bar{\beta}|^2 = 1$$

$$r_{BA}^2 P_B |\mathbf{h}_{BA}\mathbf{v}_B|^2 = r_{BA}^2 P_B |\bar{\beta}|^2 \|\mathbf{h}_{BA}\|^2 \leq J$$

then

$$\|\bar{\beta}\|^2 \leq \frac{J}{r_{BA}^2 P_B \|\mathbf{h}_{BA}\|^2}.$$

Moreover,

$$\begin{aligned} |\mathbf{h}_{BB}\mathbf{v}_B|^2 &= \lambda_{BB} \left|\mathbf{v}_B^H \mathbf{m}_0\right|^2 \\ &= \lambda_{BB} \left|(\bar{\alpha}\mathbf{v}_B^{IC} + \bar{\beta}\mathbf{u}_0)^H \mathbf{m}_0\right|^2 \\ &= \lambda_{BB} \left|\frac{\bar{\alpha}(1 - \|\mathbf{m}_0^H \mathbf{u}_0\|^2)}{\sqrt{(1-\rho_B^2)}} + \bar{\beta}\mathbf{u}_0^H \mathbf{m}_0\right|^2 \end{aligned} \tag{3.27}$$

and

$$\left|\mathbf{m}_0^H \mathbf{u}_0\right|^2 = \left|\frac{\mathbf{h}_{BB}^H \mathbf{h}_{BA}}{\left\|\mathbf{h}_{BB}^H\right\| \left\|\mathbf{h}_{BA}\right\|}\right|^2 = \rho_B^2.$$

Without losing generality, let $\bar{\alpha} \in \mathbb{R}$ and $0 \leq \bar{\alpha} \leq 1$. In order to maximise the signal power of user B, define

$$\bar{\beta} = \beta \frac{\mathbf{m}_0^H \mathbf{u}_0}{\left|\mathbf{m}_0^H \mathbf{u}_0\right|} \quad \text{and} \quad \beta \in \mathbb{R}, \quad 0 \leq \beta \leq 1$$

Then (3.26) becomes

$$\begin{aligned} \mathbf{v}_B^{IC} &= \sqrt{1-\beta^2}\mathbf{v}_B^{IF} + \beta \frac{\mathbf{m}_0^H \mathbf{u}_0}{\left\|\mathbf{m}_0^H \mathbf{u}_0\right\|}\mathbf{u}_0 \\ &= \sqrt{1-\beta^2}\mathbf{v}_B^{IF} + \beta \frac{\mathbf{h}_{BB}\mathbf{h}_{BA}^H}{\rho_B}\mathbf{h}_{BA}^H \end{aligned} \tag{3.28}$$

and (3.27) becomes

$$\begin{aligned} \left|\mathbf{h}_{BB}\mathbf{v}_B\right|^2 &= \lambda_{BB}\left|\bar{\alpha}\sqrt{1-\rho_B^2} + \beta\rho_B\right|^2 \\ &= \left\|\mathbf{h}_{BB}\right\|^2 \left|\sqrt{(1-\beta^2)(1-\rho_B^2)} + \beta\rho_B\right|^2. \end{aligned}$$

Then if

$$\rho_B \leq \sqrt{\frac{J}{r_{BA}^2 P_B \left\|\mathbf{h}_{BA}\right\|^2}}$$

we have

$$\beta = \rho_B$$

and

$$\left|\mathbf{h}_{BB}\mathbf{v}_B^{IC}\right|_{\max}^2 = \left\|\mathbf{h}_{BB}\right\|^2$$

else

$$\beta = \sqrt{\frac{J}{r_{BA}^2 P_B \left\|\mathbf{h}_{BA}\right\|^2}}$$

and

$$\left|\mathbf{h}_{BB}\mathbf{v}_B^{IC}\right|_{\max}^2 = \left\|\mathbf{h}_{BB}\right\|^2 \left|\sqrt{(1-\beta^2)(1-\rho_B^2)} + \beta\rho_B\right|^2.$$

□

From (3.23) and (3.24) we can see that allowing some interference to the primary system will increase the desired signal power of the secondary system. The key idea of the IC precoding

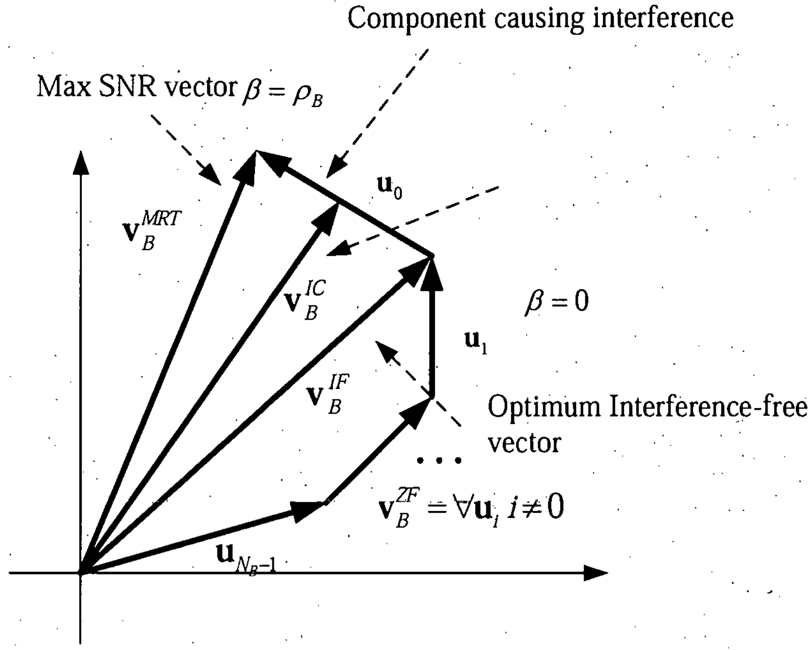


Figure 3.4: *Geometric interpretation of the relationship between single system approaches*

approach is the trade-off between the two systems. It tries to include some contributions from the component which causes the interference to the primary system. The performance of the IC precoding approach depends on the cross-correlation coefficient of the interference channel vector and the message channel vector. When the interference constraint reaches a certain value, which equals the interference level caused by the MRT method, the maximum desired received power for user B is obtained. The IC method then becomes the MRT method. When the interference limits continue to increase, both the received data signal power and the interference to the primary system will not increase. On the other hand, when the permitted interference tends to zero, the IC tends to the IF method and β tends to zero.

3.4 Comparison of single system approaches

We introduced the single system precoding approaches, MRT, ZF, IF, and IC in the four previous sections. The comparison of these approaches is presented in Table 3.1. The geometric relationships of the ZF, MRT, IF, and IC methods are illustrated in Figure 3.4. Note that for ease of understanding, the vector solutions have been drawn with different lengths in Figure 3.4. In

	Interference power to primary system	Message channel power of secondary system (G)	Required channel state information
MRT	$r_{BA}^2 \rho_B^2 \|\mathbf{h}_{BA}\|^2 P_B$	$\|\mathbf{h}_{BB}\|^2 P_B$	Message channel
ZF	Zero	$\leq G_{IF}$	Interference channel
IF	Zero	$(1-\rho_B^2)\|\mathbf{h}_{BB}\|^2 P_B$	Message and interference channel
IC	$\leq J$	$G_{IF} \leq G_{IC} \leq G_{MRT}$	Message and interference channel

Table 3.1: *Comparison of single system approaches*

the implementation of these algorithms, however, these vectors would be normalised. The MRT approach achieves the best performance according to the message channel information. Its precoding vector can be seen as the optimal linear combination in the complex vector space $\mathbb{C}^{N_B \times 1}$. The basis vector set is the set of eigenvectors of the interference self-correlation matrix. Arbitrary vectors corresponding to the zero eigenvalues represent solutions of ZF. If the component $\mathbf{u}_0$ is removed, the remaining vector of the MRT precoding vector is the optimal interference-free solution. Moreover, the interference-constrained solution is similar to the IF solution but adds part of the vector $\mathbf{u}_0$, depending on the allowable interference limits. The parameter, β, called the interference coefficient, will determine the interference and desired signal power. When $\beta = 0$, there is no interference to primary system, and the algorithm becomes equal to IF; when $\beta = \rho_B$, the algorithm becomes equal to MRT. Another thing to notice is that all the precoding methods are affected by the cross-correlation coefficient of the interference channel and the data channel. When $\rho_B = 0$, the MRT, IF, and IC become the same precoding method because no interference will be produced to the primary system. When ρ_B increases, the performance of the IF and IC, in terms of the desired signal power, will degrade and the interference caused by MRT to the primary system increases. In the worst situation, when the value of $\rho_B = 1$, for MRT and IC the interference power to primary system and the data power to desired system are equal. For the IF and ZF cases, both the interference power and the signal power are equal to zero.

3.5 Simulation

The simulations and the results of single system beamforming are presented in this section. In these simulations both System A (the primary system) and System B (the secondary system) are single user systems and only downlink scenario is considered. Section 3.5.1 gives the simulation configuration and channel conditions. The detailed simulation results and analysis are presented in Section 3.5.2.

3.5.1 Channel model and simulation conditions

We now introduce two kinds of channel models that are used in our simulations, a Rayleigh fading channel and a more practical cellular channel model, the single cell channel.

For the Rayleigh fading channel, the elements of the channel vectors are i.i.d. complex Gaussian random variables with zero mean and unit covariance. It is used to measure the bit error ratio (BER) and MSE.

The single cell channel model [53], includes Rayleigh fading, path-loss, and shadow fading. The channel vectors can be written as:

$$\mathbf{h} = \sqrt{\mathrm{SNR}_0 \left(\frac{d}{R_{cell}}\right)^{-\alpha} 10^{\delta/10}}\ \mathbf{g} \tag{3.29}$$

where SNR_0 is the median signal noise ratio when the distance between terminal and base station equals R_{cell} (the cell radius); α is the path loss exponent; δ models shadowing and is a real Gaussian random variable with zero mean and variance of σ_δ^2; d is the distance between MS and BS; g is a complex Gaussian distributed random vector variable representing Rayleigh fading with zero mean and unit variance. The single cell channel is used to measure the Shannon capacity. In the following of this thesis, assume that the received interference is Gaussian distrusted. Therefore, the Shannon capacity is (3.30)

$$C = \log_2(1 + \mathrm{SINR}) \quad (\mathrm{bits/s/Hz}). \tag{3.30}$$

We extend the single cell channel to two coexisting cell systems, as shown in Figure 3.5. A simulator for two coexisting radio systems based on C\C++ has been developed, which is used for performance simulations in this thesis. The parameters for single cell channel are defined

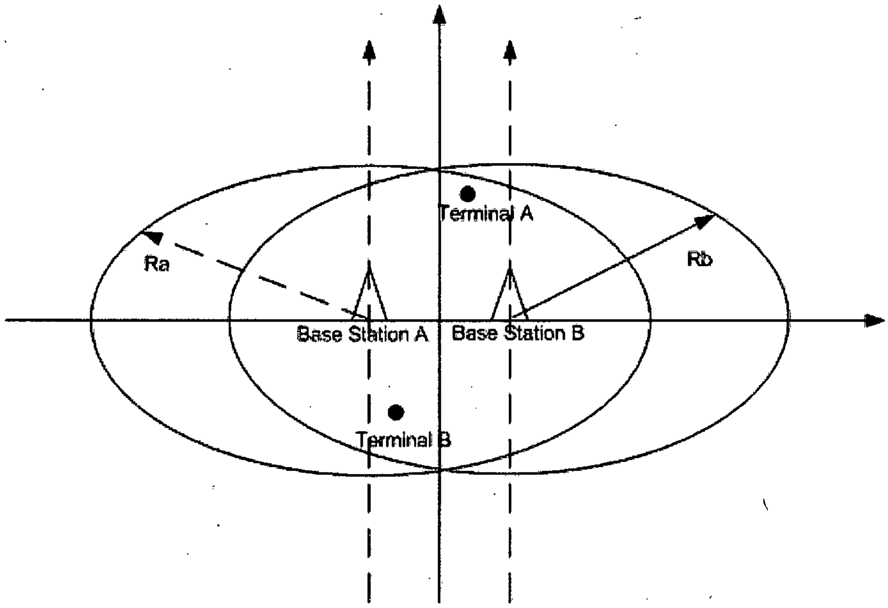


Figure 3.5: *Two coexisting cells system layout*

in Table ??. For the single system beamforming, the two cell system use the same spectrum with an overlapping cell area. Both the cell radii are 2.5km, and the distance between two base stations is 1km. For the two systems, the path loss exponent and standard deviation of shadowing is 3.4 and 4.0dB respectively. The terminals of both systems are randomly distributed in the cell areas. For both channel models, the noise in the receivers is set to 1.0 (0dB), and the acceptable interference for the primary system J is 20% of noise power. In the following discussion, the transmission power is defined as a relative value (dB) and the reference value is the noise power.

Radius ($Radius_A$ and $Radius_B$)	1.0km
Derivative of shadowing (σ)	4.0dB
Path loss	3.4
BS-BS distance	1.0km
Radius of movable areas for receivers Rs (dB)	2.5km

Table 3.2: *Parameters of single cell channel for single system beamforming*

We also assume all the channels are the slow-fading wireless channels with packet-based transmission and are quasi-static over one packet length. For one simulation, a total of 2×10^6 packets are sent, and there are 10 symbols in each packet. When measuring BER performance, both radio systems send uncoded QPSK signals. The simulation conditions for single system

	Primary system A	Secondary system B
Channel gain (dB)	0	0
Interference factor	0.5	0.5, 0.8, 1.0
Number of antenna	2	3, 4, 5
Transmission Power (dB)	10	0, 3, 5, 7, 9, 11, 13, 15

Table 3.3: *Simulation conditions for single system approaches*

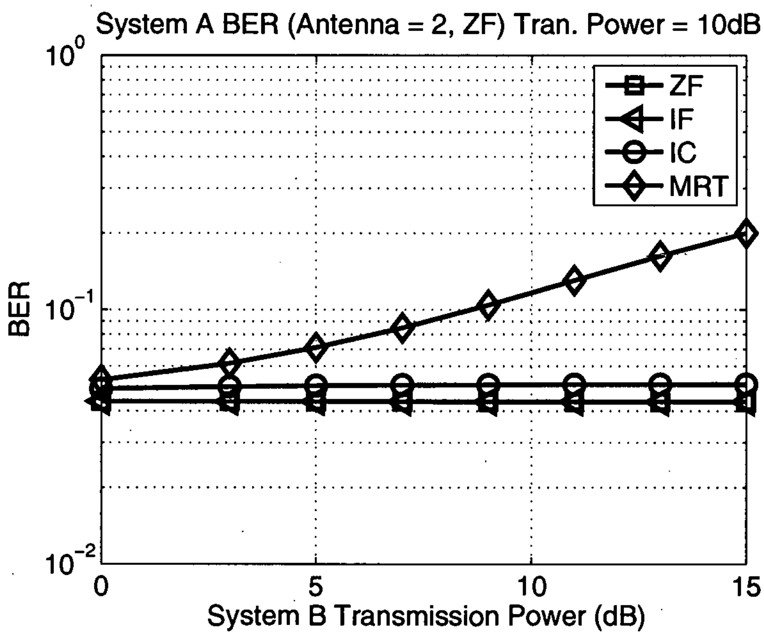


Figure 3.6: *BER of system A for ZF, IF, IC, and MRT under Rayleigh channel model (QPSK modulation)*

beamforming are listed in Table 3.3.

3.5.2 Simulation results

3.5.2.1 BER of Rayleigh channel

The BER results of system B and system A with different algorithms are shown in Figure 3.6 and Figure 3.7 respectively. From these two figures, we can see that MRT achieves the best performance for system B and causes the poorest performance for system A. This is because MRT maximises the received signals for system B (full diversity gain is obtained) and produces the serious co-channel interference to system A which greatly degrades system A's SINR.

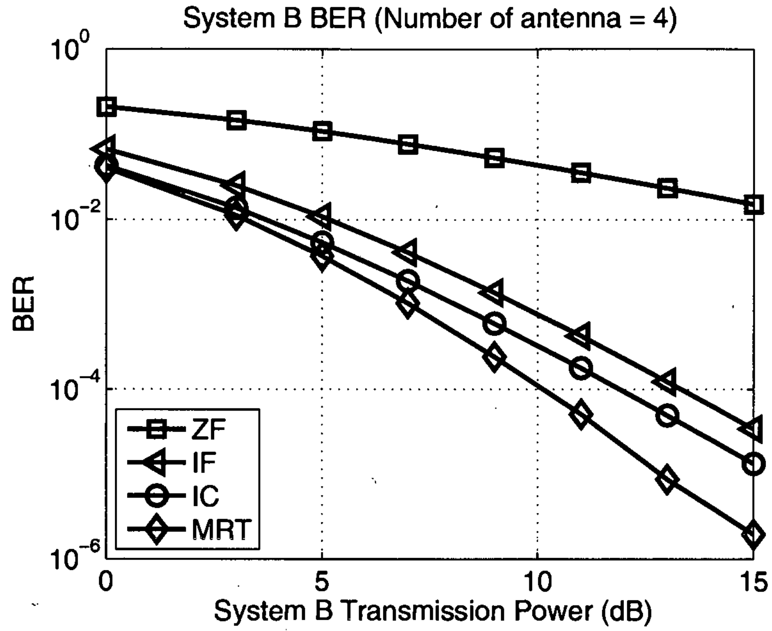


Figure 3.7: *BER of system B for ZF, IF, IC, and MRT under the Rayleigh channel model (QPSK modulation)*

The ZF method achieves the poorest performance for system B because arbitrary selection of orthogonal vector loses all the diversity gain. The performance of the IF method is superior to the ZF performance because the IF method selects the optimal orthogonal vector, and obtains the maximum achievable diversity. It is easy to understand why IF and ZF yield the same performance for system A because they do not produce any interference to it.

The performance of the IC method for system B is in-between that of the IF and the MRT method. This is because the IC method utilises part of the interference vector to achieve a higher SNR. That also is why there is a gap between the IC and the IF method for the performance of system A in Figure 3.6. Since the constraint is defined as the allowable interference power, which is directly proportional to the transmission power, the interference coefficient β for IC is inversely proportional to the transmission power. Higher transmission power corresponds to a lower coefficient β and vice versa. Moreover, a higher interference coefficient β leads to the IC performance tending to that of MRT, and a lower interference coefficient leads to an IC solution tending towards that of the IF. It explains why the IC performance tends to that of MRT when transmission power is low, and tends to that of ZF when transmission power is high.

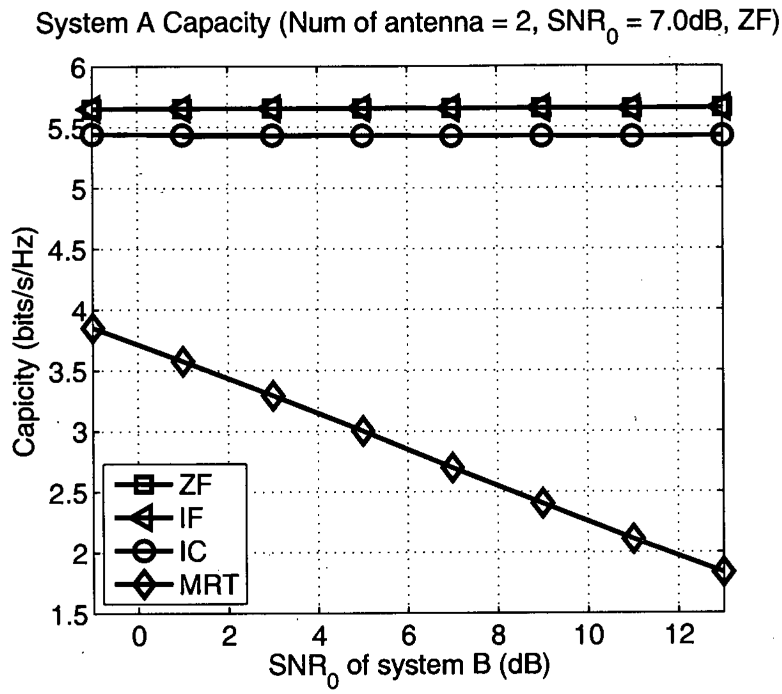


Figure 3.8: *Capacity of system A for ZF, IF, IC, and MRT under single cell channel*

3.5.2.2 Capacity of single cell channel

The capacities of system A and system B with different algorithms are shown in Figure 3.8 and Figure 3.9 respectively. It shows the same trends as with the BER measurements for a Rayleigh fading channel shown in Figure 3.6 and Figure 3.7. That is, improving the performance of system B comes at the cost of degrading the capacity of system A. The IF method achieves the best performance when there is no interference allowed to the primary system. The IC method increases the performance compared to IF when some interference is allowed by the primary system since it is the optimal combination of the IF and the MRT method.

3.5.2.3 Performance effects of system configuration and channel condition

It is interesting to consider what will happen when we increase the number of antennas. The BER comparison of the IF and the MRT for equipment with different numbers of antennas is shown in Figure 3.10. In this simulation, we compared the BER performance of system B for MRT and IF when it employs 3, 4, and 5 antennas in base station under Rayleigh fading channel. It is clear that the IF effectively loses the benefit of one antenna compared with the MRT. It

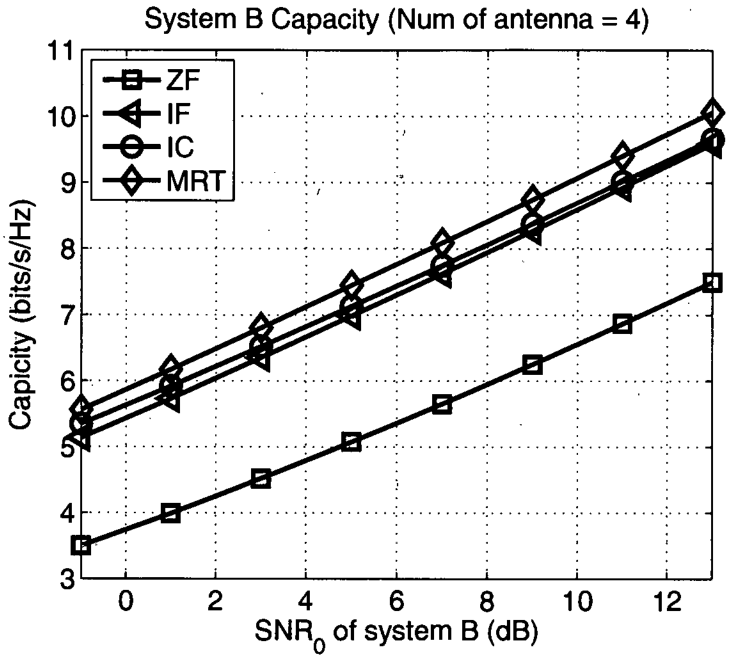


Figure 3.9: *Capacity of system B for ZF, IF, IC, and MRT under single cell channel*

is because the lost degree of freedom is used to remove interference to the primary system, following the theoretical analysis in [61]. It also verifies that the IF achieves the maximum achievable diversity subject to the interference constraint.

Furthermore, we would like to see what will happen if interference factor r changes. Figure 3.11 and Figure 3.12 show the performance comparison of IF and IC when the interference channel weight equals to 0.5, 0.8, 1.0 respectively. We can see that there is no influence on the performance of IF when the interference channel weight increases. This is because the IF only cares about the direction of interference but not the amplitude. The performance of IC has a slight decrease when the interference channel weight increases. It is because the increase of interference weight will lead to the increase of interference power and decrease of the interference coefficient if the interference constraint J is fixed. Then the performance of the IC will move towards that of the IF. In Figure 3.12 this increase causes a slight degradation in BER performance for the secondary system when IC is used.

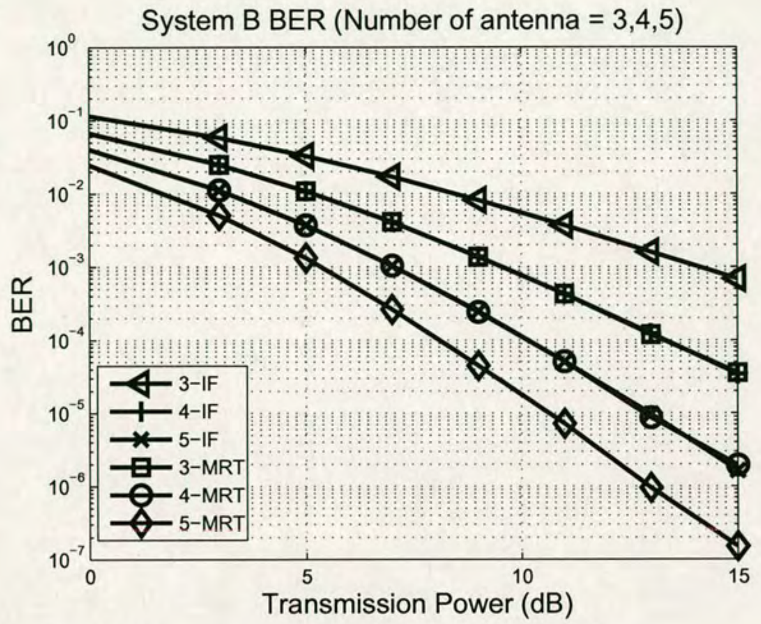


Figure 3.10: *BER of system B for IF and MRT with 3, 4, 5 antennas (Rayleigh fading channel, QPSK modulation)*

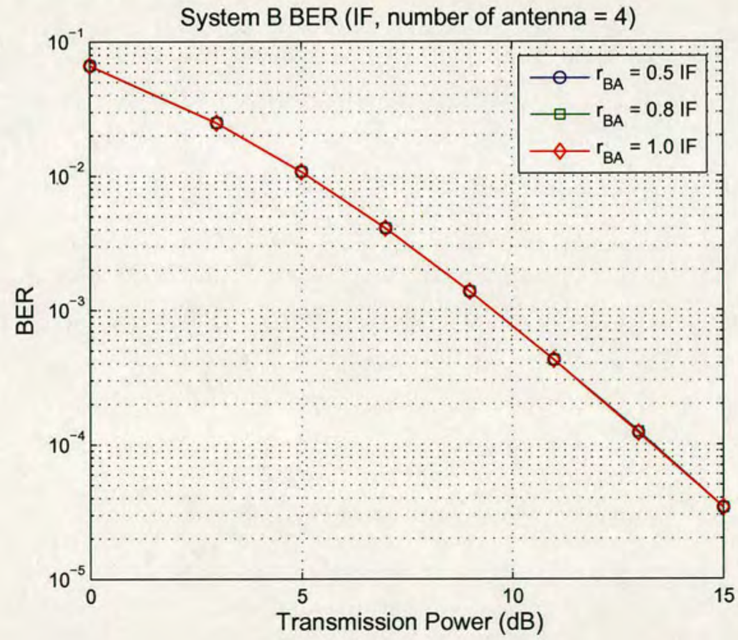


Figure 3.11: *BER of system B for IF with different interference channel factor (Rayleigh fading channel, QPSK modulation)*

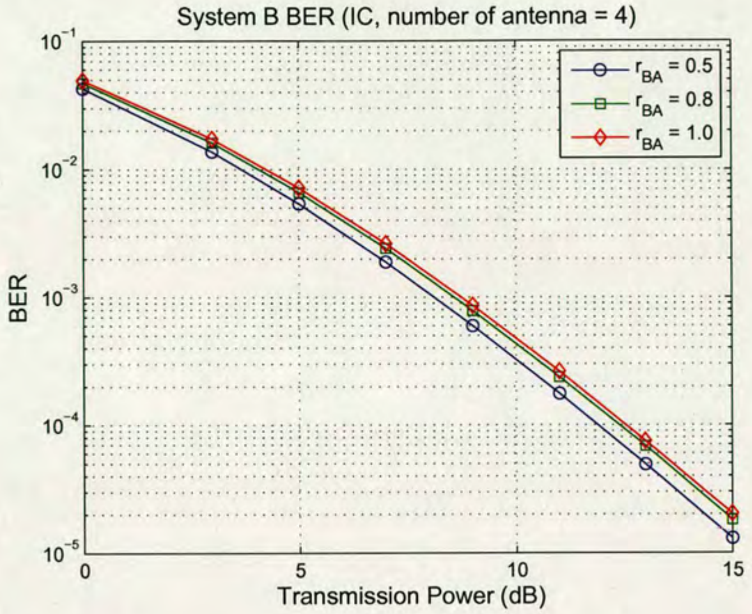


Figure 3.12: *BER of system B for IC with different interference channel factor (Rayleigh fading channel, QPSK modulation)*

3.6 Conclusion

Cognitive radio is an important solution to the lack of spectrum resource. Interference mitigation is likely to be a key technology for coexisting systems. In this chapter we discussed the precoding method for single system scenario where one system does not know the channel state information at the other. The maximum ratio transmission (MRT) and zero forcing (ZF) techniques are two well-known single system approaches. Based on this, we present and prove the optimal interference free (IF) precoding vector, which is the optimal combination of ZF precoding vectors. It cancels the interference to other system and obtains the remaining diversity gain. When the primary system can afford some interference due to the good channel conditions, we propose an interference-constrained (IC) precoding method to utilise this headroom to improve the performance of the secondary system. The IC precoding vector is the optimal combination of IF vector and an interference vector, and it can achieve the best performance for the secondary system with a primary system interference constraint. It allows a trade-off between the two systems according to quality of service (QoS) requirements of the primary system. The comparison of these single system approaches via analysis and simulation gives us a clear view of their performance and relationship.

Chapter 4
Joint Beamforming

The beamforming technologies have been proved to have capability to mitigate or control co-channel interference in coexisting environments for single system methods in the previous chapter. In this chapter, joint beamforming methods, which involve the cooperation between two coexisting radio systems, are discussed. Our major contributions in this chapter include: 1) propose adaptive joint beamforming methods which increase the performance of the secondary system but not essentially influence the performance of the primary system and is used in scenario where the coexisting radio systems have different priorities; 2) joint beamforming methods are presented and analysed in a scenario where both coexisting radio systems have the same right to access the spectrum.

The rest of this chapter is organised as follows. An introduction which answers the questions of why joint beamforming precoding methods are needed and what is the focus in this chapter is presented in Section 4.1. Then the system model and basic assumptions for joint beamforming under cognitive radio are described and discussed in Section 4.2. The adaptive joint beamforming approaches and overall joint beamforming technologies are presented in Section 4.3 and Section 4.4 respectively. Section 4.5 contains the numerical results. Finally, we conclude this chapter in Section 4.6.

4.1 Introduction

Consider the downlink of a cognitive radio environment with two coexisting radio systems. With multiple antennas in the transmitter side, beamforming techniques can be used to counteract co-channel interference and hence improve system performance. Normally, for cognitive radio, there are two types of beamforming approaches - single system beamforming and joint beamforming.

For single system beamforming, the two coexisting radio systems precode independently and do not consider the channel conditions of the other system. In the previous chapter, several single system beamforming approaches have been discussed and compared, including maximum ratio transmission (MRT), zero-forcing (ZF), optimal interference free precoding (IF), and optimal interference constrained precoding (IC). The MRT achieves the best performance for the secondary system, but degrades the primary system due to interference. The ZF achieves the poorest performance for the secondary system while not interfering with the primary system using only the interference-channel state information. The IF is superior to ZF with both data and interference channel state information. The performance of IC is between the MRT and the IF, while the primary system suffers controlled interference. However, although these precoding methods can avoid or control the co-channel interference, they cannot adapt according to the channel of the coexisting system and hence may unacceptably degrade the required performance of the primary system or unnecessarily reduce the performance of the secondary system. For example, when IC is used, the transmitter of the secondary system will induce a fixed interference to the primary system. If the channel of the primary system is very good, the interference from the secondary system may actually be too small and there will still be some headroom to increase the secondary system performance. Moreover, if the channel of the primary system is bad, the interference from the secondary system may be the vital factor to cause the required quality of service (QoS) to be violated.

The other beamforming approach for cognitive radio is joint beamforming. This method precodes with knowledge of channel state information (CSI) of both systems and therefore can adapt according to different channel conditions. It can avoid the problems of single system methods, and may achieve optimal performance for the overall cognitive radio system (both radio systems) at the cost of increasing computational complexity and extra exchange of channel state information.

Our focus in this chapter is on finding joint beamforming precoding solutions for the downlink of multiple-input single-output cognitive radio (MISO-CR) systems. We consider two scenarios:

- One system has priority to access the spectrum and needs to achieve some specified QoS;
- The two systems have same right to access the spectrum, and we consider the overall optimisation of system performance.

For the first scenario, an adaptive method to let the secondary system use the spectrum while not affecting the primary system is proposed. A low computational complexity (LCC) variant with little performance loss is also presented. For the second scenario, we propose beamforming methods according to the criteria of sum mean squared error (SMSE), continuous sum throughput, and discrete sum throughput. The simulation results show that the proposed precoding algorithms can maximise the utilisation of multiple antennas and improve the system performance.

4.2 System model

Considering the MISO-CR system, a similar system model to the previous chapter is used. Also suppose that there are two radio systems, system A and system B. System A is the primary system and system B is the secondary system. The only difference to Chapter 3 is what information is fed back to the transmitters. In joint beamforming methods, the receivers estimate their corresponding data channels and interference channel information, and feed back the channel state information to the both transmitters. The transmitters will use this information as they compute the beamforming vectors.

Similar to Chapter 3, the received signals y_A and y_B for the lth sample time can be written as:

$$\begin{aligned} y_A(l) &= \mathbf{h}_{AA}(l)\mathbf{v}_A(l)s_A(l) + r_{BA}\mathbf{h}_{BA}(l)\mathbf{v}_B(l)s_B(l) + z_A(l) && (4.1)\\ y_B(l) &= \underbrace{\mathbf{h}_{BB}(l)\mathbf{v}_B(l)s_B(l)}_{\mathrm{I}} + \underbrace{r_{AB}\mathbf{h}_{AB}(l)\mathbf{v}_A(l)s_A(l)}_{\mathrm{II}} + \underbrace{z_B(l)}_{\mathrm{III}} && (4.2) \end{aligned}$$

The total transmitter power constraint for system A and system B are denoted by $\mathrm{E}[|s_A(l)|^2] \leq P_A$ and $\mathrm{E}[|s_B(l)|^2] \leq P_B$. The constraints for the beamforming vectors are $\|\mathbf{v}_A\|^2 \leq 1$ and $\|\mathbf{v}_B\|^2 \leq 1$.

Furthermore, since an arbitrary linear precoding vector $\mathbf{v}$ for a MISO-CR system can be seen as the sum of an interference component and an orthogonal component (Chapter 3) [62], we can use the interference power to represent the precoding vectors. Define the interference coefficient as the square root of the ratio of the average interference power to the maximum possible interference power for a given channel configuration. It is easy to understand this definition is equal to the interference coefficient defined by Chapter 3. The interference coefficients of

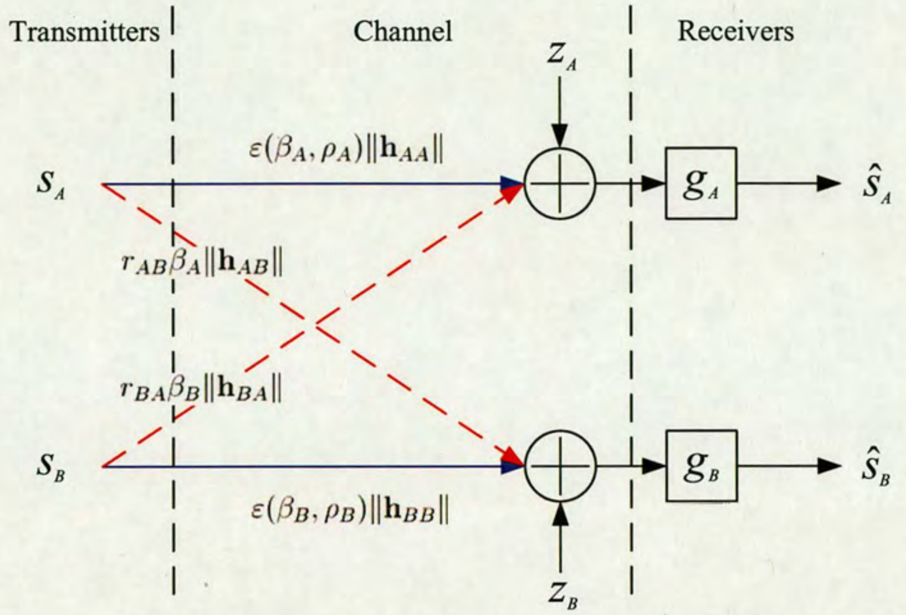


Figure 4.1: *Cognitive radio channel model with beamforming*

system A, β_A, and system B, β_B, are shown in following equations,

$$\beta_{\mathrm{A}} = \sqrt{\frac{\mathrm{E}[\,\|\mathbf{h}_{AB}\mathbf{v}_A s_A\|^2]}{\mathrm{E}[\,\|\mathbf{h}_{AB} s_A\|^2]}} = \frac{\|\mathbf{h}_{AB}\mathbf{v}_A\|}{\|\mathbf{h}_{AB}\|} \tag{4.3}$$

$$\beta_{\mathrm{B}} = \frac{\|\mathbf{h}_{BA}\mathbf{v}_B\|}{\|\mathbf{h}_{BA}\|} \tag{4.4}$$

where $0 \le \beta_A, \beta_B \le 1$. Then the interference power from system A and B, denoted as I_A and I_B respectively, can be presented as:

$$I_A = r_{AB}^2 \beta_A^2 \|\mathbf{h}_{AB}\|^2 P_A \tag{4.5}$$

$$I_B = r_{BA}^2 \beta_B^2 \|\mathbf{h}_{BA}\|^2 P_B \tag{4.6}$$

According to Theorem 3.2 in Chapter 3 (Theorem 2 in reference [62]), for any given interference constraint, there exists an optimal linear precoding vector that achieves the maximum data channel gain. Moreover, the maximum data channel gain is a function of its interference coefficient when the channel state information is fixed. So the cognitive radio channel model can be represented as in Figure 4.1.

The function $\varepsilon(\beta,\rho)$ which is related to the data channel gain for a given β is [62] (Chapter 3):

$$\varepsilon(\beta,\rho)=\sqrt{(1-\beta^2)(1-\rho^2)}+\beta\rho \tag{4.7}$$

where ρ is the cross-correlation coefficient, defined as follows for system A and B:

$$\rho_A=\frac{\|\mathbf{h}_{AB}\mathbf{h}_{AA}^H\|}{\|\mathbf{h}_{AB}\|\,\|\mathbf{h}_{AA}\|} \quad \text{and} \quad \rho_B=\frac{\|\mathbf{h}_{BA}\mathbf{h}_{BB}^H\|}{\|\mathbf{h}_{BA}\|\,\|\mathbf{h}_{BB}\|} \tag{4.8}$$

When the channels are fixed, the cross-correlation coefficients are constant, so we use the simplified notation $\varepsilon_A(\beta_A)$ and $\varepsilon_B(\beta_B)$ to indicate $\varepsilon(\rho_A,\beta_A)$ and $\varepsilon(\rho_B,\beta_B)$ respectively. Then the received signals power for system A and B, denoted as G_A and G_B respectively, are follows:

$$G_A = \varepsilon_A^2(\beta_A)\,\|\mathbf{h}_{AA}\|^2 P_A \tag{4.9}$$

$$G_B = \varepsilon_B^2(\beta_B)\,\|\mathbf{h}_{BB}\|^2 P_B. \tag{4.10}$$

Therefore, the MISO-CR channel is simplified to a "cognitive interference channel", where the interference channel and the data channel can affect each other via adjusting the interference factors. When the precoding vectors are changed, the interference to the other system may increase or decrease depending on the requirements. Moreover, it offers an opportunity to obtain the best trade-off between coexisting radio systems to achieve the optimal system performance.

According to equations (4.1), (4.2), (4.5), (4.6), (4.9) and (4.10), when the channel is fixed, the signal-to-interference-plus-noise ratios (SINRs) for system A and system B are the functions of the interference coefficients β_A and β_B. They can be presented as:

$$\mathrm{SINR_A}(\beta_A,\beta_B) = \frac{G_A}{I_B+\sigma_A^2}=\frac{\varepsilon_A^2(\beta_A)\,\|\mathbf{h}_{AA}\|^2 P_A}{r_{BA}^2\beta_B^2\,\|\mathbf{h}_{BA}\|^2 P_B+\sigma_A^2} \tag{4.11}$$

$$\mathrm{SINR_B}(\beta_A,\beta_B) = \frac{G_B}{I_A+\sigma_B^2}=\frac{\varepsilon_B^2(\beta_B)\,\|\mathbf{h}_{BB}\|^2 P_B}{r_{AB}^2\beta_A^2\,\|\mathbf{h}_{AB}\|^2 P_A+\sigma_B^2} \tag{4.12}$$

The signal-to-noise-ratio (SNR) at the receivers only considers thermal noise, which is equal to the SINR when the interference of the other radio system equals zero.

$$\mathrm{SNR_A}\left(\beta_A\right) = \mathrm{SINR_A}\left(\beta_A, 0\right) = \frac{\varepsilon_A^2\left(\beta_A\right)\left\|\mathbf{h}_{AA}\right\|^2 P_A}{\sigma_A^2} \tag{4.13}$$

$$\mathrm{SNR_B}\left(\beta_B\right) = \mathrm{SINR_B}\left(\beta_B, 0\right) = \frac{\varepsilon_B^2\left(\beta_B\right)\left\|\mathbf{h}_{BB}\right\|^2 P_B}{\sigma_B^2} \tag{4.14}$$

4.3 Adaptive joint beamforming

The spectrum is allocated to the primary radio system by a government organisation (e.g. OFCOM or FCC), so normally it has higher priority to access this resource. Any other radio system which wants to utilise the licensed spectrum, should not affect the primary radio system performance or at least ensure the primary radio system performs beyond its required QoS. The QoS may correspond to the rate, bit error ratio (BER), or SINR in the physical layer.

Here we consider such a MISO-CR scenario. Assume that both systems have fixed transmission power. The primary system tries its best to achieve an indicated SINR via beamforming while the secondary system maximises its performance subject to the constraint of meeting the required SINR of the primary system. Since we consider the fading channel, the primary system cannot always achieve the target SINR even if there is no other radio systems to share the spectrum. However, we can use the statistical characteristics of the SINR as our metric. If it has not essentially changed, we can say that the secondary system does not affect the performance of the primary system.

However, how do we define the "essential" change of statistical characteristics for the SINR? From the users' opinion, they do not care about physical layer parameters. If the QoS is achieved, they do not think an SINR of 12dB is better than an SINR of 11dB. Moreover, if the basic QoS is not acceptable, for example, they cannot connect or lots of interference exists due to the fading channel and the environment, they also do not think an SINR of 2dB is worse than an SINR of 3dB. They only care about how often the data or voice services does not operate satisfactorily. Based on this, we can use the probability of the SINR being greater than a target value, which indicates the QoS is satisfied, and the probability of the SINR being less than another target value, which indicates the QoS is not acceptable as our standards. If these probabilities have not changed for the given scenario when the secondary system is introduced, we can say that the secondary system does not affect the performance of the primary system.

Keeping this in mind, we can easily find a way to ensure the secondary system does not affect the performance of primary system when they share the spectrum. Our idea is as follows:

- When the channel of the primary system is very good, improve the performance of the secondary system while keeping the QoS of primary system above the required standard;
- When the channel of the primary system is very bad, the secondary system does not need to consider the primary system and the primary system avoids interfering with the secondary system;
- For other situations, the secondary system avoids interfering with the primary system.

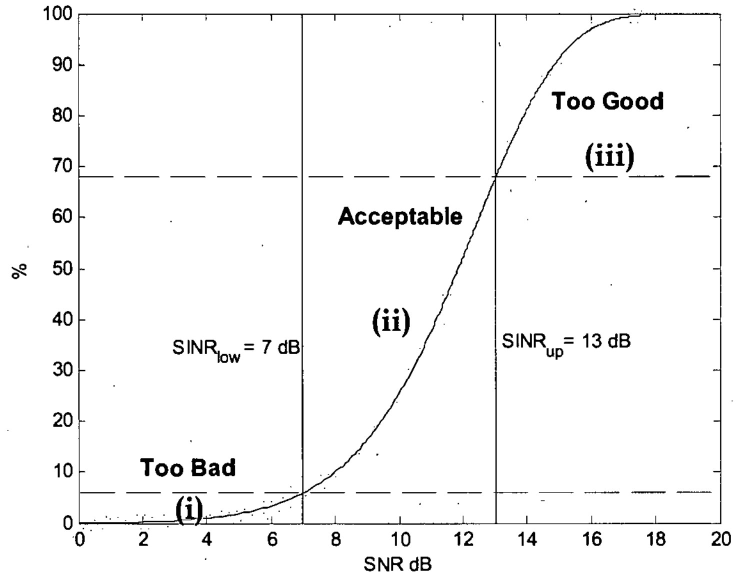


Figure 4.2: *CDF of SNR for primary system under Rayleigh channel fading when MRT is used*

We define SINR_{up} and SINR_{low} to indicate the channel conditions and the QoS. These two parameters divide the SINR of the primary system into three parts, as shown in Figure 4.2. This figure shows the cumulative distribution function (CDF) of SNR for a three-antenna radio system with MRT beamforming, unit power Rayleigh channel fading, unit noise power, and 10dB transmission power. Here, $\text{SINR}_{\text{up}} = 13\text{dB}$ and $\text{SINR}_{\text{low}} = 7\text{dB}$. If $\text{SINR}_{\text{A}} > \text{SINR}_{\text{up}}$, the area (iii) in Figure 4.2, the channel is too good, and the secondary system can use this headroom while keeping the primary system SINR at an acceptable value. If $\text{SINR}_{\text{A}} < \text{SINR}_{\text{low}}$, the area (i) in the figure, the channel is too bad to transmit signals, so the secondary system can use the spectrum freely. When $\mathrm{SINR_{low}} \leq \mathrm{SINR_A} \leq \mathrm{SINR_{up}}$, the area (ii) in Figure 4.2, the channel of the primary system is just acceptable, and the secondary system needs to avoid interfering with the primary system.

The task can be represented as:

$$\begin{aligned} &\{\beta_A^{Ad};\, \beta_B^{Ad}\} = \arg\max_{\{\beta_A,\beta_B\}}\{\ \mathrm{SINR_B}\} \\ &s.t. \quad \Pr\{\mathrm{SINR_A} \geq \mathrm{SINR_{up}}\} = \max \Pr\{\mathrm{SINR_A} \geq \mathrm{SINR_{up}} | \beta_B = 0\} \\ &\qquad\ \Pr\{\mathrm{SINR_A} \leq \mathrm{SINR_{low}}\} = \min \Pr\{\mathrm{SINR_A} \leq \mathrm{SINR_{low}} | \beta_B = 0\} \end{aligned} \tag{4.15}$$

where $\mathrm{SINR_A}$ indicates the QoS requirement of the primary system. Obviously, when the MRT is used in system A, the probability of $\mathrm{SINR_A} \geq \mathrm{SINR_{up}}$ is maximised and probability of $\mathrm{SINR_A} \leq \mathrm{SINR_{low}}$ is minimised for an arbitrary channel distribution when there is no co-channel interference.

Here we give a solution of this problem for the MISO-CR environments. We consider it for different channel conditions, just as in Figure 4.2.

- When $\mathrm{SNR_A^{MRT}} \leq \mathrm{SINR_{low}}$, which means

$$\frac{\|\mathbf{h}_{AA}\|^2 P_A}{\sigma_A^2} \leq \mathrm{SINR_{low}},$$

  we do not need to consider the performance of the primary system. So to maximise the $\mathrm{SINR_B}$, the primary system uses the any method without interference to the secondary system, such as the IF or the ZF, and the secondary system uses the MRT method[1];

- When $\mathrm{SINR_{low}} \leq \mathrm{SNR_A^{MRT}} \leq \mathrm{SINR_{up}}$, this means that the primary system should not be interfered. So the MRT method is used for the primary system, and the IF method is used for the secondary system;

- When $\mathrm{SNR_A^{MRT}} \geq \mathrm{SINR_{up}}$, the secondary system can use the headroom until $\mathrm{SINR_A}$ approaches $\mathrm{SINR_{up}}$. According to equations (4.11) and (4.12), this problem can be

[1]For multiple users scenario, if one of the primary system users has very poor channel conditions, this resource may be used for other users first. Only when all the active users of the primary system satisfies this condition, the secondary system user can use MRT on this resource

rewritten as:

$$\left\{\beta_A^{Ad};\ \beta_B^{Ad}\right\} = \underset{\{\beta_A,\beta_B\}}{\arg\max}\left\{\frac{\varepsilon_B^2(\beta_B)\left\|\mathbf{h}_{BB}\right\|^2 P_B}{r_{AB}^2\beta_A^2\left\|\mathbf{h}_{AB}\right\|^2 P_A+\sigma_B^2}\right\}$$

$$s.t.\ \frac{\varepsilon_A^2(\beta_A)\left\|\mathbf{h}_{AA}\right\|^2 P_A}{r_{BA}^2\beta_B^2\left\|\mathbf{h}_{BA}\right\|^2 P_B+\sigma_A^2} \geq \mathrm{SINR}_{\mathrm{up}},\quad 0\leq\beta_A\leq\rho_A\,\mathrm{and}\,0\leq\beta_B\leq\rho_B \quad (4.16)$$

Due to the non-linearity of $\varepsilon(\beta)$, it is very difficult to find a closed-form solution of this problem. We can use a non-linear search method to obtain the optimal solution.

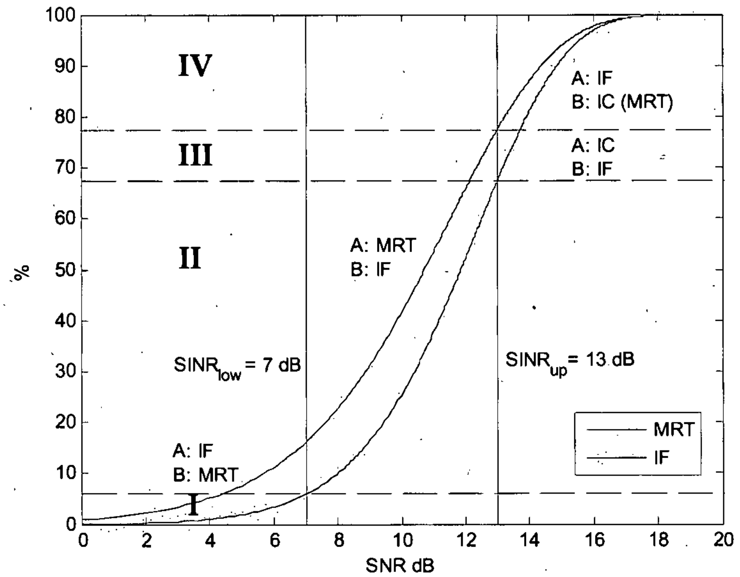


Figure 4.3: *Low computational complexity adaptive joint beamforming under Rayleigh channel*

To decrease the computational complexity and avoid iterative methods to get a solution, we also propose a sub-optimal beamforming method here. We believe that the high SNR case is more important than low SNR case because it can supply good service and the interference normally has more effect on the SINR than the channel gain when the SNR is high. Therefore, in the low computational complexity (LCC) method, we try to use beamforming methods that can avoid or generate as little as possible interference to the other system. We divide the SINR into four parts, just shown in Figure 4.3. The area (I) and (II) are the same as the area (i) and (ii) as in Figure 3. We further divide the area (iii) in Figure 4.2 into two parts. In this follows, $\mathrm{SNR}_{\mathrm{A}}^{\mathrm{MRT}}$ and $\mathrm{SNR}_{\mathrm{A}}^{\mathrm{IF}}$ denote the SNRs of the primary system when the MRT and the IF methods are used for the primary system with no co-channel interference respectively.

- When $\mathrm{SNR}_{\mathrm{A}}^{\mathrm{MRT}} \leq \mathrm{SINR}_{\mathrm{low}}$, the LCC solution is the same as the optimal solution, where the IF is used for the primary system and the MRT is used for the secondary system;
- When $\mathrm{SINR}_{\mathrm{low}} \leq \mathrm{SNR}_{\mathrm{A}}^{\mathrm{MRT}} \leq \mathrm{SINR}_{\mathrm{up}}$, just as in part (ii) in Figure 4.2, the MRT is used for the primary system and the IF is used for the secondary system;
- When $\mathrm{SNR}_{\mathrm{A}}^{\mathrm{IF}} \geq \mathrm{SINR}_{\mathrm{up}}$, the part (IV) in Figure 4.3, which means

$$\frac{\left(1-\rho_A^2\right)\left\|\mathbf{h}_{AA}\right\|^2 P_A}{\sigma_A^2} \geq \mathrm{SINR}_{\mathrm{up}},$$

according to equations (4.13) and (4.2), the IF method is used by the primary system. Then the interference that the primary system can afford is:

$$I_B = \frac{\left(1-\rho_A^2\right)\left\|\mathbf{h}_{AA}\right\|^2 P_A}{\mathrm{SINR}_{\mathrm{up}}} - \sigma_A^2 \tag{4.17}$$

The IC method is used for the secondary system. According to equations (4.5) and (4.6), we have

$$\beta_B^{Ad} = \min\left\{\rho_B, \tilde{\beta}\right\} \tag{4.18}$$

where

$$\tilde{\beta} = \frac{\sqrt{\frac{\left(1-\rho_A^2\right)\left\|\mathbf{h}_{AA}\right\|^2 P_A}{\mathrm{SINR}_{\mathrm{up}} P_B} - \frac{\sigma_A^2}{P_B}}}{r_{BA}\left\|\mathbf{h}_{BA}\right\|} \tag{4.19}$$

- When $\mathrm{SNR}_{\mathrm{A}}^{\mathrm{IF}} < \mathrm{SINR}_{\mathrm{up}} < \mathrm{SNR}_{\mathrm{A}}^{\mathrm{MRT}}$, the secondary system uses the IF method to avoid interference to the primary system, and the primary system uses the IC method to increase the channel gain, and the received SNR. The interference factor β_A is:

$$\beta_A^{Ad} = \rho_A\sqrt{\frac{\mathrm{SINR}_{\mathrm{up}}\sigma_A^2}{\left\|\mathbf{h}_{AA}\right\|^2 P_A}} - \sqrt{\left(1-\rho_A^2\right)\left(1-\frac{\mathrm{SINR}_{\mathrm{up}}\sigma_A^2}{\left\|\mathbf{h}_{AA}\right\|^2 P_A}\right)} \tag{4.20}$$

In addition, the interference to the secondary system is :

$$I_A = r_{AB}^2 \left\|\mathbf{h}_{AB}\right\|^2 \left[P_A \left(1-\rho_A^2\right) + \left(2\rho_A^2 - 1\right) \frac{\mathrm{SINR_{up}}\sigma_A^2}{\left\|\mathbf{h}_{AA}\right\|^2} \right.$$
$$\left. -2\rho_A\sqrt{1-\rho_A^2}\frac{\sqrt{\mathrm{SINR_{up}}\sigma_A^2\left(\left\|\mathbf{h}_{AA}\right\|^2 P_A - \mathrm{SINR_{up}}\sigma_A^2\right)}}{\left\|\mathbf{h}_{AA}\right\|^2} \right] \tag{4.21}$$

Proof. According to the previous discussion, the IC method should be used for the primary system and IF method is used for the secondary system ($\beta_B = 0$).Therefore, from equation (4.11), the $\mathrm{SINR_A}$ is following:

$$\mathrm{SINR_A} = \mathrm{SINR_A}(\beta_A^{Ad}, 0) = \frac{\varepsilon_A^2(\beta_A^{Ad})\|\mathbf{h}_{AA}\|^2 P_A}{\sigma_A^2} \tag{4.22}$$

Moreover, since we try to match $\mathrm{SINR_A}$ to $\mathrm{SINR_{up}}$, which means

$$\mathrm{SINR_A}(\beta_A^{Ad}, 0) = \mathrm{SINR_{up}} \tag{4.23}$$

Then, according to equation (4.2), (4.22), and (4.23), and , the value of β_A^{Ad} and interference from the primary system to the secondary system I_A can be obtained with the constraints $0 \leq \beta_A^{Ad} \leq \rho_A$, which are shown in equations (4.20) and (4.21). □

4.4 Overall joint beamforming

In the previous section, we discussed the case where one radio system has higher priority. This case usually occurs in licensed spectrum, and that is why the performance of the primary system must be satisfied. The situation changes in unlicensed spectrum. All radio systems have the same right to use this radio resource. So for this case, the aim becomes how to achieve overall optimal performance for all coexisting radio systems.

For the MISO-CR environment, the optimisation for global performance usually needs to consider both systems together, and the two systems need to be aware of each other. From equations (4.2), (4.5), (4.6), (4.9), and (4.10), when the interference to the other system increases via increasing the interference coefficients for $\beta \leq \rho$, the desired system's signal power also increases. (For $\rho < \beta \leq 1$, the increase of interference to the other system lead to the decrease of the desired system's power.) Therefore, the increase of one system's performance is likely to lead to the decrease of the other system's performance. The key point of the joint optimal method is to find the best trade-off for the two systems and to optimise the chosen criterion. In this section, possible criteria may be the maximum sum capacity and minimum sum mean squared error.

$$\begin{aligned} \left\{\mathbf{v}_A^{op};\, \mathbf{v}_B^{op}\right\} &= \underset{\{\mathbf{v}_A, \mathbf{v}_B\}}{\arg} \;\{\text{ optimal criterion value}\} \\ &s.t.\ \|\mathbf{v}_A\|^2 = \|\mathbf{v}_B\|^2 = 1 \end{aligned} \tag{4.24}$$

Moreover, these criteria are related to the interference and data signal power, which are the functions of interference coefficients. Then the issue of finding the optimal joint beamforming vectors becomes to find the optimal interference coefficients according to the chosen criterion. The joint optimal problem can be represented as:

$$\begin{aligned} \left\{\beta_A^{op};\, \beta_B^{op}\right\} &= \underset{\{\beta_A, \beta_B\}}{\arg} \;\{\text{ optimal criterion value}\} \\ \text{s.t.:}\;\; & 0 \leq \beta_A \leq \rho_A \;\text{ and }\; 0 \leq \beta_B \leq \rho_B \\ & \mathbf{v}_A^{op} = \mathbf{v}_A(\beta_A^{op}) \;\text{ and }\; \mathbf{v}_B^{op} = \mathbf{v}_B(\beta_B^{op}) \end{aligned} \tag{4.25}$$

In the rest of this section, we will discuss joint beamforming methods for two important criteria - sum mean squared error (SMSE) and sum throughput.

4.4.1 Sum mean squared error

Usually the mean squared error (MSE) can be seen as the indicator of BER, which is a very important parameter to estimate the performance of a radio system. Providing that the MSE is minimised for radio systems, the minimum BER can often be achieved. Here in MISO-CR scenarios, we try to minimise the weighted sum normalised mean squared error (SMSE) to optimise the overall performance of both coexisting radio systems.

The normalised minimum mean squared error (MMSE) is a function of the interference coefficients for a given channel condition. From the discussion in Chapter 3 and equations (4.5), (4.6), (4.2), (4.9), and (4.10) the MMSEs for both coexisting radio systems under the MISO-CR environment are:

$$\mathrm{MMSE}_A^{nr}(\beta_A,\beta_B)=\frac{r_{BA}^2P_B\beta_B^2\left\|\mathbf{h}_{BA}\right\|^2+\sigma_A^2}{P_A\varepsilon_A^2(\beta_A)\left\|\mathbf{h}_{AA}\right\|^2+r_{BA}^2P_B\beta_B^2\left\|\mathbf{h}_{BA}\right\|^2+\sigma_A^2} \tag{4.26}$$

$$\mathrm{MMSE}_B^{nr}(\beta_A,\beta_B)=\frac{r_{AB}^2P_A\beta_A^2\left\|\mathbf{h}_{AB}\right\|^2+\sigma_B^2}{P_B\varepsilon_B^2(\beta_B)\left\|\mathbf{h}_{BB}\right\|^2+r_{AB}^2P_A\beta_A^2\left\|\mathbf{h}_{AB}\right\|^2+\sigma_B^2}. \tag{4.27}$$

Define the weighted sum mean squared error (SMSE) as

$$\mathrm{SMSE}(\beta_A,\beta_B)=w\mathrm{MMSE}_A^{nr}(\beta_A,\beta_B)+(1-w)\mathrm{MMSE}_B^{nr}(\beta_A,\beta_B) \tag{4.28}$$

where $w\in[0,1]$ is a weight that indicates which system is more important in terms of performance optimisation. A larger weight means a higher priority. For example, if $w=1/2$, the two systems have same priority; if $w=1$ or $w=0$, the MSE of system B or system A is totally ignored. According to equation (4.25), the problem can be presented as:

$$\{\beta_A^{op};\beta_B^{op}\}=\underset{\{\beta_A,\beta_B\}}{\arg}\ \min\mathrm{SMSE}(\beta_A,\beta_B)$$

$$s.t.\ 0\le\beta_A\le\rho_A \text{ and } 0\le\beta_B\le\rho_B \tag{4.29}$$

The closed-form solution for minimising the SMSE is usually difficult to obtain because of the non-linearity of the $\varepsilon(\beta)$ function. Furthermore, the SMSE is not always a convex function of the interference coefficients in the search region. So only local minima may be found via standard search methods, and it is very difficult to determine whether a given stationary point is the globally best solution. The obtained solution may be worse than the interference free (IF) method (Chapter 3) [62] where the interference coefficients equal zero. Here we give an example of a strong interference case to show that SMSE is not a convex function in this scenario. Assume that $\rho_A=\rho_B=0.8$, $P_A\|\mathbf{h}_{AA}\|^2=20$, $r_{AB}^2P_A\|\mathbf{h}_{AB}\|^2=35$, $P_B\|\mathbf{h}_{BB}\|^2=0.15$, $r_{BA}^2P_B\|\mathbf{h}_{BA}\|^2=1.5$, and $\sigma_A^2=\sigma_B^2=1.0$, then the SMSE for $0\le\beta_A\le\rho_A$ and $0\le\beta_B\le\rho_B$ is shown in Figure 4.4 . It is clear that there are two minima in the search area.

For the low SNR case, the performance will not improve significantly when increasing the interference coefficient because of the high noise power. The difference of performance between

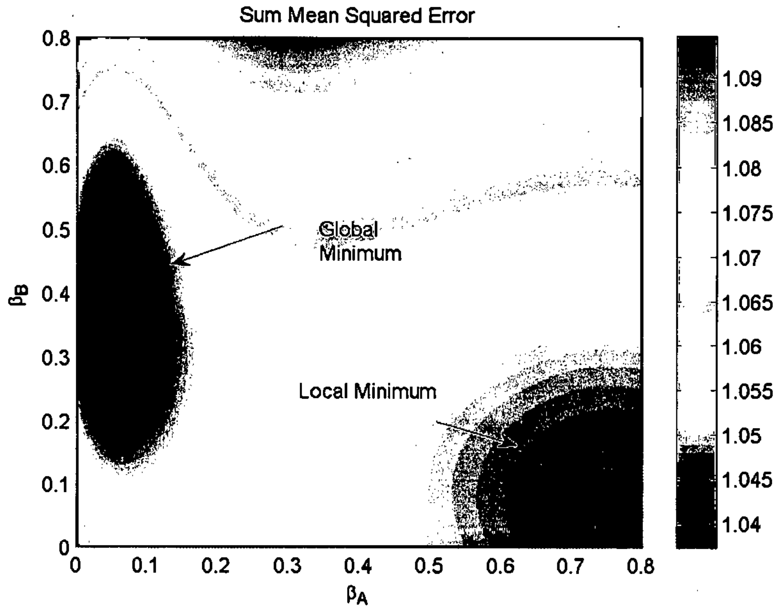


Figure 4.4: *An example for two minima in the search area for SMSE*

the optimal solution and IF method is small. So using the IF method for such situation, although it is not the best solution, will not lose too much performance. For the high SNR case, the interference power from the other system will greatly influence the MSE because the noise power tends to zero. The optimal solution is usually near the IF point for high SNR case. For example, when the noise power equals to zero, the optimal solution is $\beta_A = \beta_B = 0$, which is named the interference-free-and-interference-free precoding (IF-IF) method. Therefore, the IF will be a good selection if we can not find the optimal solution. According to the discussion, here we propose a sub-optimal joint beamforming scheme which outperforms the IF method to find the solution of minimising equation (4.28). The key idea of our method is:

- Using an iterative method to find the minimum with initial point $\beta_A^0 = \beta_B^0 = 0$ (i.e. IF-IF);
- If the performance of the found solution is poorer than the IF method, use that instead of the found solution.

This solution is not the optimal solution, but it can at least provide performance better than the IF-IF method, so the performance will improve. This is called the sub-optimal joint beamforming solution.

The iterative steps are presented as follows:

1. Let $\beta_A^{(0)} = \beta_B^{(0)} = 0$; define ϱ and ς, where ϱ is the convergence condition and ς is the search precision. Compute $\mathrm{SMSE}(\beta_A, \beta_B)$;

2. Fix $\beta_B^{(n)}$, find the local minimum with precision ς; if $\mathrm{SMSE}(\beta_A^{(n+1)}, \beta_B^{(n)})$ is greater than $\mathrm{SMSE}(0, \beta_B^{(n)})$, $\beta_A^{(n+1)} = 0$;

3. Fix $\beta_A^{(n+1)}$, find the local minimum with precision ς; if $\mathrm{SMSE}(\beta_A^{(n+1)}, \beta_B^{(n+1)})$ is greater than $\mathrm{SMSE}(\beta_A^{(n+1)}, 0)$, $\beta_B^{(n+1)} = 0$;

4. Let $\Delta m = \mathrm{SMSE}(\beta_A^{(n)}, \beta_B^{(n)}) - \mathrm{SMSE}(\beta_A^{(n+1)}, \beta_B^{(n+1)})$, if $\Delta m \leq \varrho$ goto 5, else goto Step 2;

5. $\mathbf{v}_A^{op} = \mathbf{v}_A^{IC}(\beta_A^{op})$ and $\mathbf{v}_B^{op} = \mathbf{v}_B^{IC}(\beta_B^{op})$.

The computational complexity of this method is about equal to a two-dimensional search optimisation problem, but it depends on which optimisation methods are used. In here, step 2 and step 3 are both one-dimensional optimisations. The general line search methods, such as Golden Section and Fibonacci methods [63], which are not hugely complex to implement, can solve this problem. This iterative algorithm does not need to be applied to the situation where the cross-correlation coefficient of interference channel and data channel $\rho = 0$. If $\rho = 0$, the corresponding system can use the MRT method directly because it will not affect the other system due to channel orthogonality.

4.4.2 Sum throughput

Another important parameter for radio systems is throughput, which determines how fast the radio system can transmit and what service it can supply. In this subsection, we discuss the sum continuous throughput and sum discrete throughput for MISO-CR with joint beamforming respectively, and the latter is more important practically since we assume only an integral number of bits can be sent in a symbol. This is representative of realistic systems that can only transmit using a finite set of modulation and coding configurations.

4.4.2.1 Sum continuous throughput

When M-QAM modulation is used and the error probability is measured in terms of SER, the number of bits per symbol R can be presented as [64] [65]

$$R = \log\left(1 + \frac{\text{SINR}}{\Gamma}\right) \tag{4.30}$$

where the Γ is the so-called "SNR Gap" and SINR is the signal-to-interference-noise-ratio. According to equations (4.11), (4.12), and (4.30) the sum throughput (SUM-R) for MISO-CR system can be written as:

$$\text{SUM–R}\left(\beta_A, \beta_B\right) = R_A\left(\beta_A, \beta_B\right) + R_B\left(\beta_A, \beta_B\right) \tag{4.31}$$

where

$$R_A\left(\beta_A, \beta_B\right) = \log\left(1 + \frac{P_A \varepsilon_A^2\left(\beta_A\right)\|\mathbf{h}_{AA}\|^2}{\Gamma_A\left(r_{BA}^2 P_B \beta_B^2\|\mathbf{h}_{BA}\|^2 + \sigma_A^2\right)}\right) \tag{4.32}$$

$$R_B\left(\beta_A, \beta_B\right) = \log\left(1 + \frac{P_B \varepsilon_B^2\left(\beta_B\right)\|\mathbf{h}_{BB}\|^2}{\Gamma_B\left(r_{AB}^2 P_A \beta_A^2\|\mathbf{h}_{AB}\|^2 + \sigma_B^2\right)}\right) \tag{4.33}$$

Both systems have an independent SNR gap Γ_A and Γ_B respectively, and that means the two systems may have different modulation and coding schemes and SER requirements. According to equation (4.25), the problem can be presented as:

$$\left\{\beta_A^{op}; \beta_B^{op}\right\} = \underset{\{\beta_A, \beta_B\}}{\arg} \quad \max \text{SUM–R}(\beta_A, \beta_B)$$

$$s.t.\ 0 \leq \beta_A \leq \rho_A \text{ and } 0 \leq \beta_B \leq \rho_B \tag{4.34}$$

This equation is similar to equation (4.28), Therefore, it has the same problems - no closed-form solution and the cost function may not be convex in the search region. Here we also give an example to show that SUM-R is not a convex function in this scenario. Assume that $\rho_A = \rho_B = 0.8$, $P_A\|\mathbf{h}_{AA}\|^2 = 100$, $r_{AB}^2 P_A\|\mathbf{h}_{AB}\|^2 = 100$, $P_B\|\mathbf{h}_{BB}\|^2 = 10$, $r_{BA}^2 P_B\|\mathbf{h}_{BA}\|^2 = 10$, $\Gamma_A = 1.5$, $\Gamma_A = 2.0$, and $\sigma_A^2 = \sigma_B^2 = 1.0$, then the SUM-R for $0 \leq \beta_A \leq \rho_A$ and $0 \leq \beta_B \leq \rho_B$ is shown in Figure 4.5. It is clear that there are two maxima in the search area.

Moreover, it is clear that the noise power has a stronger influence than in equation (4.28) for

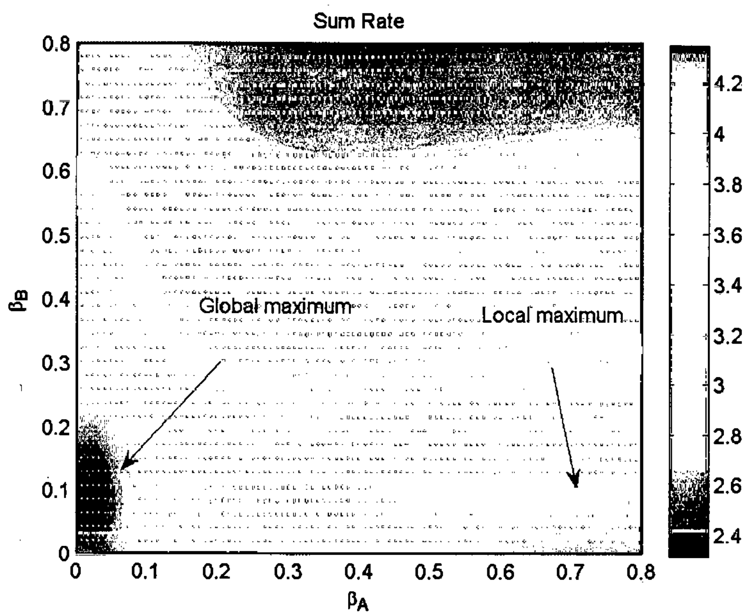


Figure 4.5: *An example for two maxima in search area for SUM-R*

the MSE criterion. When the noise power tends to zero, the optimal sum throughput tends to infinity. Therefore, the interference free beamforming still is a good selection when an optimal solution cannot be found. We can apply the sub-optimal solution that is proposed for the SMSE problem in the previous subsection to maximising SUM-R.

4.4.2.2 Sum discrete throughput

Continuous rates are difficult to implement in practice and many real systems use a finite set of code rates, so in this subsection we use a rate set which is restricted to an integer number of bits per modulation symbol. The easiest method that can ensure the required SER performance is to round the optimal continuous throughput to the nearest integers towards zero, but it may lose some data rate because both systems do not exploit their SINR sufficiently, just as shown in Figure 4.6. In this figure, the continuous solution of SUM-R is (R_A^{Op}, R_B^{Op}) and the corresponding SINRs are SINR_A^{Op} and SINR_B^{Op}. However, since the continuous rate needs to round down to a smaller integer, to reach this integer rate with given SER requirements the two systems only need SINR_A^0 and SINR_B^0 as their SINRs, the green components in this figure, respectively. Therefore, the red components show the wasted SINR for both systems.

In MISO-CR environments, we can control the SINR via adjusting the precoding vector, and

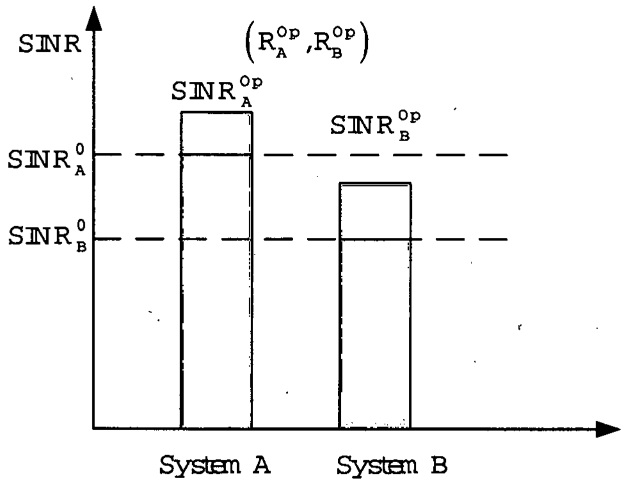


Figure 4.6: *Possible loss of SINR for discrete throughput*

this gives us the opportunity to exploit the wasted SINR. In this subsection, we use bits/channel as the unit of rate.

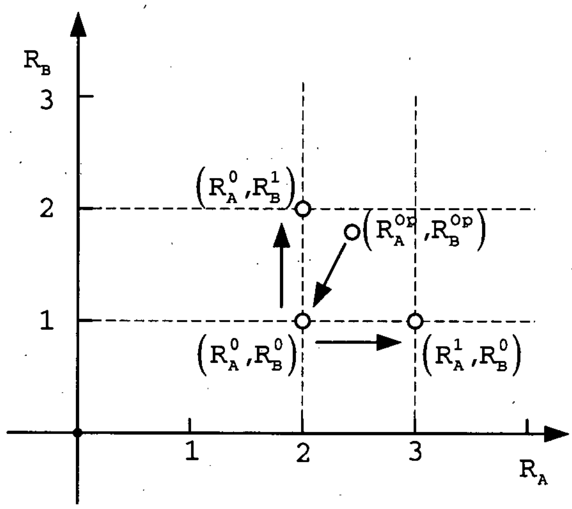


Figure 4.7: *Optimisation of discrete sum throughput for MISO-CR*

Our key idea about the optimisation of discrete sum throughput is described in Figure 4.7, and it is:

- Use the rounded results of the continuous sub-optimal method as our initial estimates for the users' data rates;
- Try to increase the SINR of one system while ensuring the throughput of the other system is no less than the rounded results, to increase the SUM-R metric.

In this figure, (R_A^{Op}, R_B^{Op}) is the continuous throughput result; (R_A^0, R_B^0) is the rounded integer result from the continuous result; (R_A^1, R_B^0) and (R_A^0, R_B^1) are two possible results of discrete throughput pairs for different directions. We select the largest achievable sum throughput as our final solution.

Assuming that for (R_A^{Op}, R_B^{Op}) the interference coefficients are $(\beta_A^{Op}, \beta_B^{Op})$, then according to equations (4.11), (4.12), and (4.30), the SINR for both coexisting systems are:

$$\mathrm{SINR_A^{Op}}\left(\beta_A^{Op}, \beta_B^{Op}\right) = \Gamma_A\left(2^{R_A^{Op}} - 1\right) = \frac{\varepsilon_A^2\left(\beta_A^{Op}\right)\|\mathbf{h}_{AA}\|^2 P_A}{r_{BA}^2\left(\beta_B^{Op}\right)^2\|\mathbf{h}_{BA}\|^2 P_B + \sigma_A^2} \tag{4.35}$$

$$\mathrm{SINR_B^{Op}}\left(\beta_A^{Op}, \beta_A^{Op}\right) = \Gamma_B\left(2^{R_B^{Op}} - 1\right) = \frac{\varepsilon_B^2\left(\beta_A^{Op}\right)\|\mathbf{h}_{BB}\|^2 P_B}{r_{AB}^2\left(\beta_A^{Op}\right)^2\|\mathbf{h}_{AB}\|^2 P_A + \sigma_B^2} \tag{4.36}$$

Here, consider finding the interference coefficients pair which maximise $\mathrm{SINR_A}$ while keep $\mathrm{SINR_B}$ above $\mathrm{SINR_B^0}$ as an example, which is clearly symmetrical to maximising the $\mathrm{SINR_B}$. The task can be presented as:

$$\begin{gathered}\left\{\beta_A^{Op}; \beta_B^{Op}\right\} = \underset{\{\beta_A, \beta_B\}}{\arg\max}\{\ \mathrm{SINR_A}\} \\ s.t.\ \frac{\varepsilon_B^2(\beta_B)\|\mathbf{h}_{BB}\|^2 P_B}{r_{AB}^2\beta_A^2\|\mathbf{h}_{AB}\|^2 P_A + \sigma_B^2} \geq \mathrm{SINR_B^0}, \quad 0 \leq \beta_A \leq \rho_A \,\mathrm{and}\, 0 \leq \beta_B \leq \rho_B\end{gathered} \tag{4.37}$$

where

$$\mathrm{SINR_B^0} = \Gamma_B\left(2^{R_B^0} - 1\right)$$

The search method has to be used to obtain the optimal solution of such problem due to the non-linearity of the $\varepsilon(\beta)$ function. However, when the new SINR is obtained, we need to recalculate the continuous throughput and round the result again, and there may still be some wasted performance. Therefore, as long as there is no great performance difference between sub-optimal and optimal schemes, the discrete throughput of both methods may be the same,

6. If $R_A^0 + R_B^1 \geq R_A^1 + R_B^0$, $(\beta_A, \beta_B) = (\beta_A^1, \beta_B^1)$; otherwise, $(\beta_A, \beta_B) = (\beta_A^2, \beta_B^2)$. Then $\mathbf{v}_A^{op} = \mathbf{v}_A^{IC}(\beta_A)$ and $\mathbf{v}_B^{op} = \mathbf{v}_B^{IC}(\beta_B)$.

4.5 Simulation

The simulation results are presented in this section. Two kinds of channels are used in our simulations: one is the Rayleigh fading channel, and the other is the single cell channel. For the Rayleigh channel, the elements of the channel vectors are i.i.d. complex Gaussian random variables with zero mean and unit covariance. The single cell channel model is same as Chapter 3, including Rayleigh fading, path-loss, and shadow fading. The detailed values of each parameter are listed in Table 4.1.

Radius ($Radius_A$ and $Radius_B$)	1.0km
Derivative of shadowing (σ)	4.0dB
Path loss	3.4
BS-BS distance	1.0km
Radius of movable areas for receivers Rs (dB)	2.5km
$r_{AB} = r_{BA}$	0.5

Table 4.1: *Simulation conditions for joint system approaches*

We also assume that the channels are slow-fading wireless channels with packet-based transmission and are quasi-static over one packet length. The transmitters for both systems employ four antennas, and the receivers employ one antenna. Both radio systems send uncoded QPSK signals, and the noise in the receivers is set to unity (0dB). In the following discussion, the transmission power is defined as a relative value (dB) and the reference value is the noise power. The simulation results of adaptive beamforming and joint overall optimal beamforming are presented in subsection 4.5.1 and 4.5.2 respectively.

4.5.1 Adaptive joint beamforming

Assume that system A is the primary system, and system B is the coexisting radio system that will share the spectrum with system A. The low bound for the primary system $\mathrm{SINR}_{\mathrm{low}} = 7\mathrm{dB}$ and the upper bound bound $\mathrm{SINR}_{\mathrm{up}} = 16\mathrm{dB}$. On reflection these value are conservative but they show the basic idea. Here, we compare the performance of the optimal adaptive beamforming, low computational complexity adaptive beamforming, and the MRT (for system A)

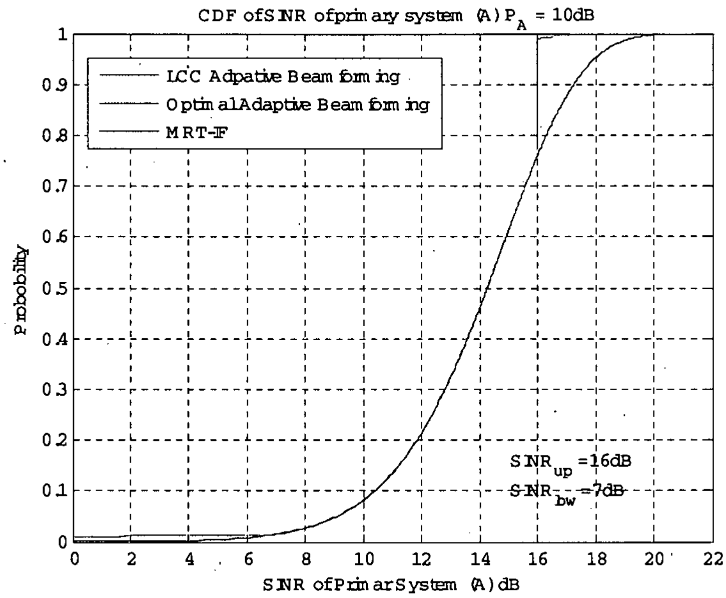


Figure 4.8: *CDF of SINR of the primary system (A) under Rayleigh channel*

IF (for system B) methods, in which the primary system achieves the best performance. The simulation is based on Rayleigh fading channel model.

The CDFs of SINR for the primary system are shown in Figure 4.8, where the transmission power of system A and system B is 10dB. From this figure, we can see that: (1) when the SINR is less than 7dB, the CDF for low computational complexity (LCC) and optimal methods increase slowly, because in this condition most SINRs of the primary system are very small due to the interference from the secondary system. (2) When the SINR is larger than 16dB, for LCC and optimal methods, the SINRs are held at 16dB because the secondary system uses this headroom to improve its performance. (3) The probability of SINR for the primary system between 7dB and 16dB for three methods are equal. So we can see that the QoS has not been affected by the secondary system.

The average capacities of the secondary system and the primary system with fixed primary system transmission power (10dB) are presented in Figure 4.9 and Figure 4.10 respectively. From Figure 4.9 it is clear that the optimal and the LCC method outperform the MRT-IF for the secondary system due to the fact that this method utilises the channel conditions of the primary system to improve the performance of the secondary system. The performance of the optimal method is only slightly better than the LCC method for lower SNRs (low transmission power),

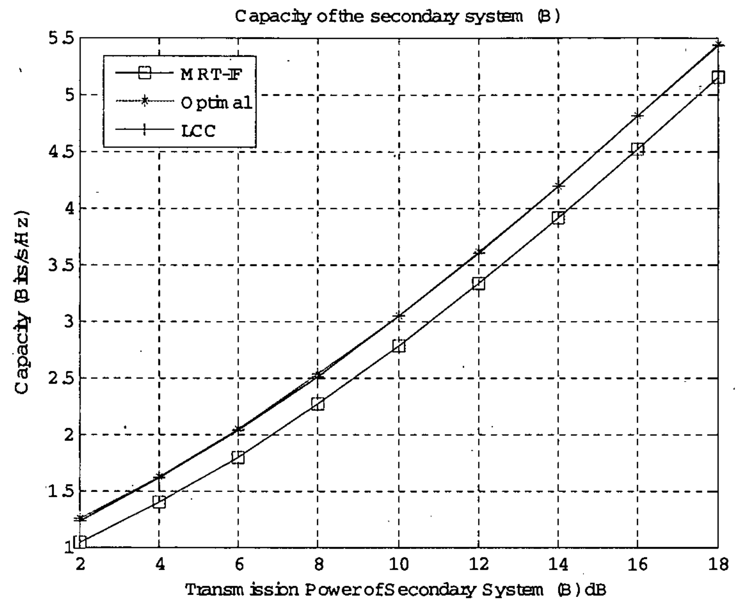


Figure 4.9: *Capacity of the secondary system (B) under Rayleigh channel*

therefore, the LCC is a quite good solution. In Figure 4.10, the capacity of the primary system for the MRT-IF is a little larger (about 0.1 bit) than the LCC method because the primary system loses some capacity when the SINR is less and greater than $\mathrm{SINR}_{\mathrm{low}}$ and $\mathrm{SINR}_{\mathrm{up}}$ respectively. When the transmission power of the secondary system increases, the average capacity of the primary system slightly decreases because the secondary system will generate more interference, however, the required QoS of the primary system will not be compromised.

4.5.2 Overall joint beamforming

In this subsection, we compare the performance of the joint overall optimisation beamforming method, including interference-free-interference-free (IF-IF), sub-optimal minimum (S-Min) sum MSE, and sub-optimal maximum (S-Max) sum throughput under the single cell channel. In these simulations, we let all SNR gaps be 1. For the continuous case, we thus compute the Shannon capacity. The transmission power for the primary system is 10 dB and the transmitters for both systems have 4 antennas for all simulations.

The sum MSEs for IF-IF and S-Min sum MSE are displayed in Figure 4.11. It clear that the S-Min sum MSE outperforms the IF-IF method. Therefore, the S-Min sum MSE should also

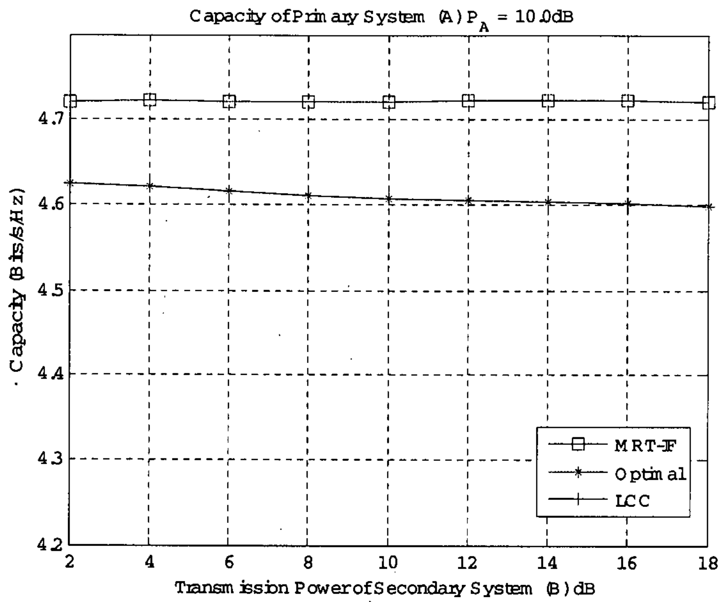


Figure 4.10: *Capacity of the primary system (A) under Rayleigh channel*

achieve the best BER performance, which is presented in Figure 4.12 and Figure 4.13. Both radio systems send uncoded QPSK signals. It is clear to see that when the transmission power of the secondary system is small, the received interference of the primary system is also small, therefore the primary system has a higher BER.

The capacity performance for these methods is presented in Figure 4.14, Figure 4.15, and Figure 4.16. Figure 4.16 shows the sum capacity performance. As we expect, the S-Max sum capacity achieves better performance than IF-IF method. However, the IF-IF method also tends to the performance of the S-Max method when the transmission power is higher. Because of its low complexity, it is also a good practical choice. The capacity performance of system A and system B are presented in Figure 4.14 and Figure 4.15 respectively.

The performance of the discrete throughput case is shown in Figure 4.17. In this figure, we compare two methods: one is the discrete results of the S-Max sum method, which rounds the results down to nearest integer toward zero, and the other is the joint discrete method. It is clear that we can get about 0.1 to 0.2 bits/s/Hz benefit when the joint discrete method is used and the benefit increases while the transmission power of system B increases. This is because when the transmission power of system B is small, if one decreases the SINR of system A, the increase of

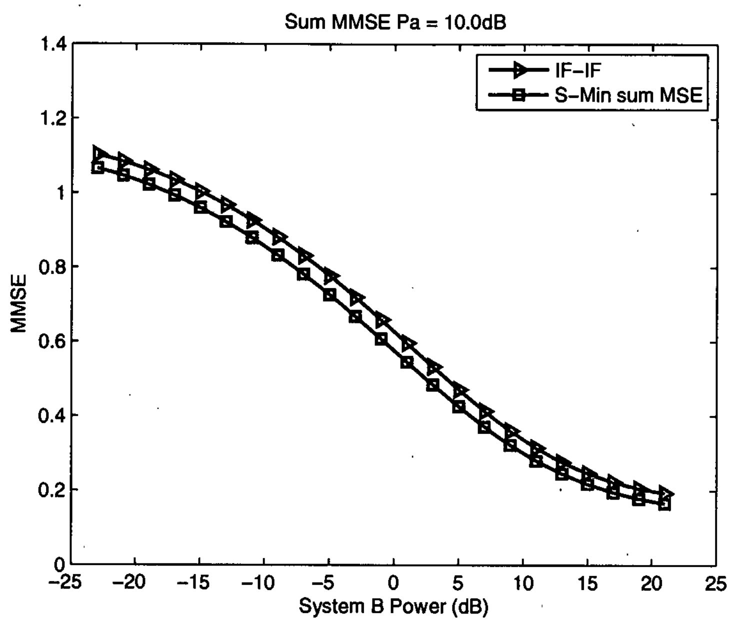


Figure 4.11: *Sum MSE under single cell channel*

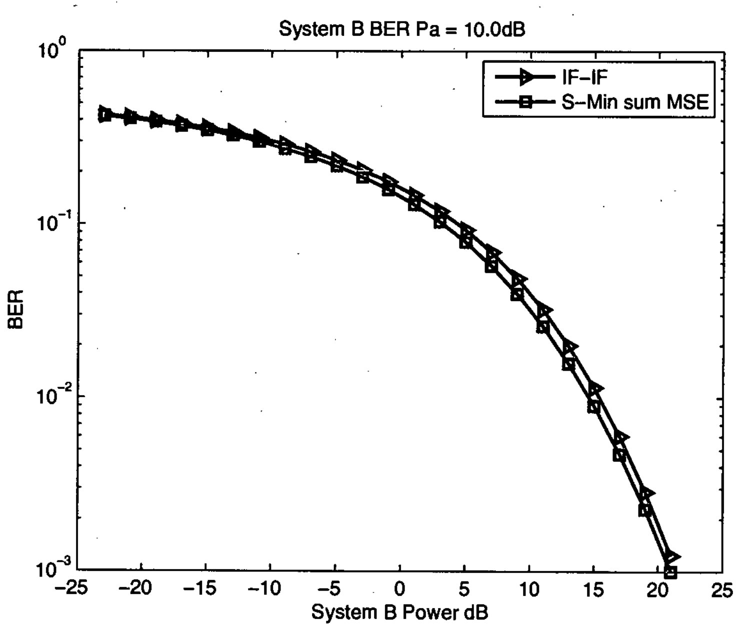


Figure 4.12: *BER of System B under single cell channel*

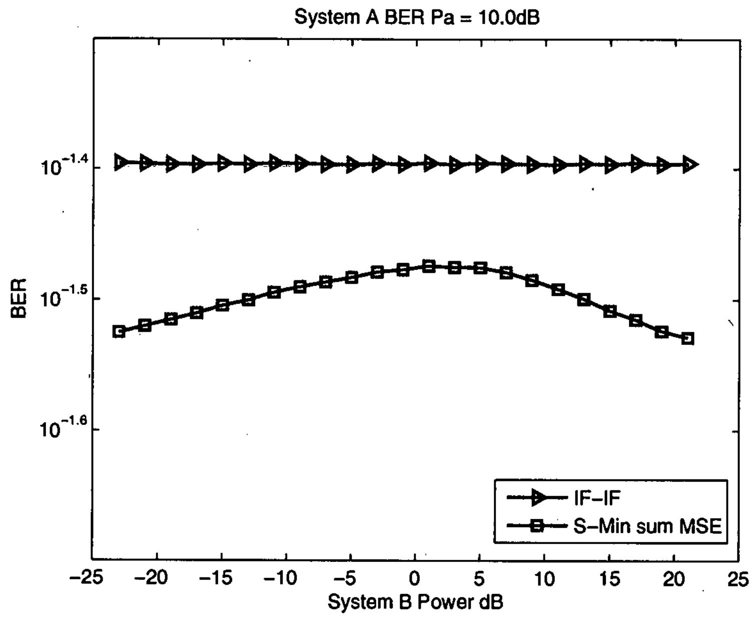


Figure 4.13: *BER of System A under single cell channel*

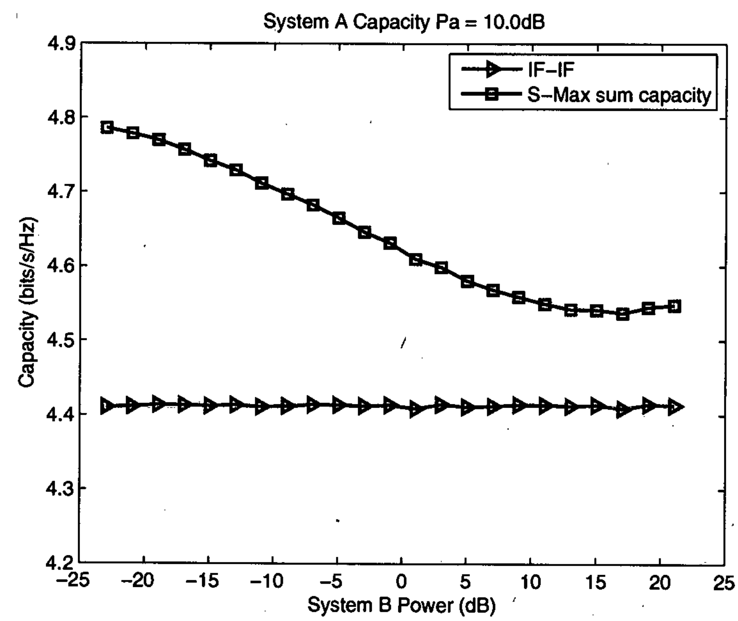


Figure 4.14: *Capacity of System A under single cell channel*

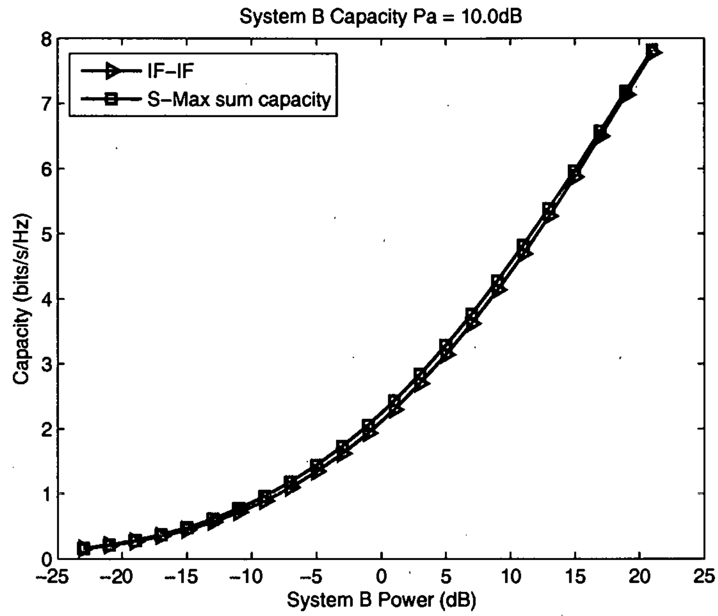


Figure 4.15: *Capacity of System B under single cell channel*

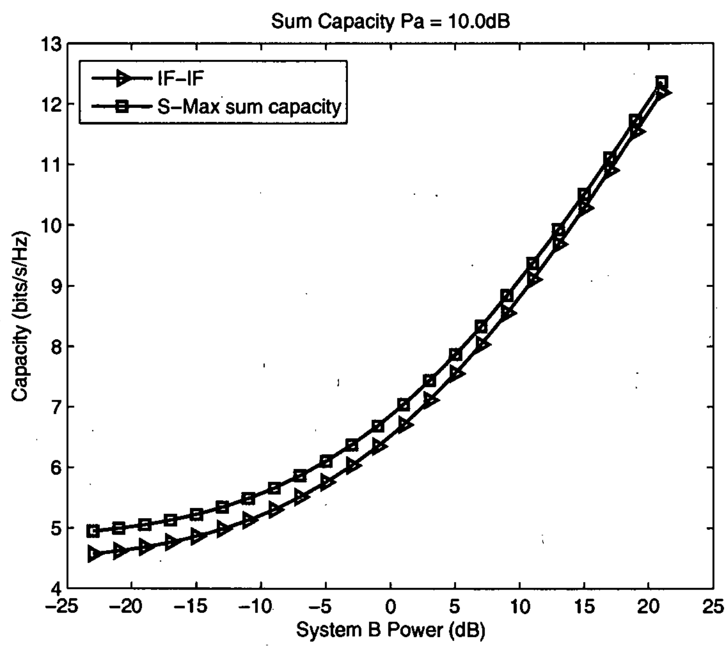


Figure 4.16: *Sum capacity under single cell channel*

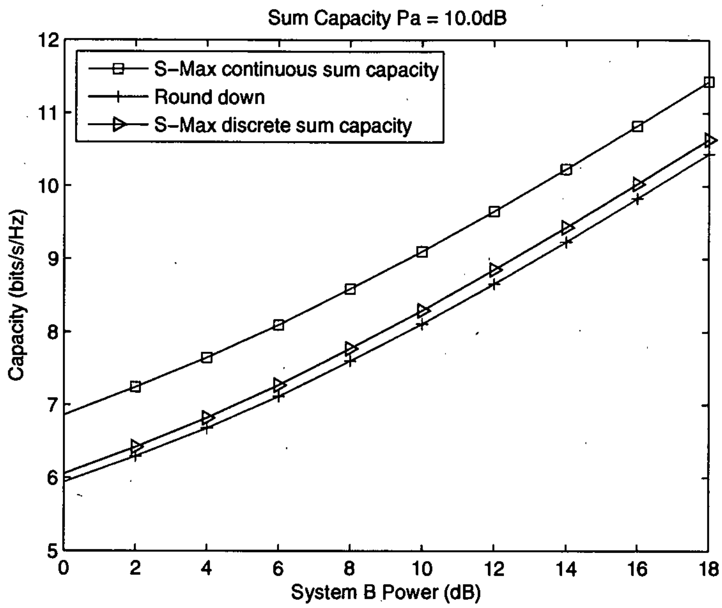


Figure 4.17: *Sum discrete capacity under single cell channel*

power of system A. So in this condition only a smaller benefit can be achieved.

4.6 Conclusion

The beamforming technique can be used in multiple-input and single-output cognitive radio (MISO-CR) systems to cancel co-channel interference. The single system beamforming methods in which radio systems independently precode according their own channel state information for the downlink of MISO-CR have been discussed in Chapter 3. However, these methods do not consider channel state information (CSI) of other coexisting systems, therefore leading to a decrease in performance. In this chapter, we discuss joint beamforming when the two radio systems perfectly know all the data and interference channel state information for multiple-input and single-output cognitive radio environments. These two systems may either have equal rights to transmit or one system has higher priority to access the spectrum and the proposed precoding algorithms achieve the best performance trade-offs between two radio systems and maximally increase the spectrum efficiency.

For the scenario where the primary system needs to achieve some specified QoS, we propose an adaptive beamforming method where the secondary system only uses the spectrum when the channel conditions of the primary system is good or bad enough. Therefore, the performance of the secondary system is improved while the QoS has not been affected for the primary system. Another method is also proposed to reduce the computational complexity, called the LCC adaptive beamforming technique. The simulation results verify that its performance is only slightly lower than the optimal method.

For the scenario of two radio systems having the same priority, we discuss beamforming methods that can achieve the best overall performance. The criteria of SMSE, continuous sum throughput, and discrete sum throughput are considered in this chapter. The simulation results show that the proposed methods usually have better performance than the IF-IF method, in which the interference-channel state information of other radio system is not known.

Chapter 5
Antenna Selection

Beamforming approaches for cognitive radio have been discussed in Chapter 3 and Chapter 4. In this Chapter, we discuss antenna selection as a method to reduce complexity. The major contributions of this chapter include: 1) analysing the single antenna selection schemes and proposing a maximum signal power to leak interference power ratio (SLIR) method to achieve the trade-off between coexisting radio systems in single antenna scenario; 2) proposing a subset optimal antennas selection scheme in multiple antennas selection scenario to reduce the complexity while keeping near optimal performance.

The rest of this Chapter is organised as follows. Section 5.1 gives a brief introduction about antenna selection; Section 5.2 describes the system model when antenna selection technologies are used in cognitive radio environments. Single antenna selection techniques are presented in Section 5.3, and multiple antennas selection technologies with single system beamforming are discussed in Section 5.4. Section 5.5 provides the simulation results for both antenna selection strategies. We conclude this chapter in Section 5.6.

5.1 Introduction

Multiple antennas is an important technology which can be used for interference mitigation. Using beamforming to cancel co-channel interference (CCI) in cognitive radio environments is one application of multiple antennas and the results shown in the previous chapters prove the benefits of multiple antennas. The single system methods include maximum ratio transmission (MRT), zero-forcing (ZF), optimal interference free (IF), and optimal interference-constrained (IC), in which the coexisting systems precode independently without the other transmitter's channel state information (CSI). For joint system methods, the coexisting systems cooperate according to the full CSI and therefore can achieve different kinds of performance trade-off between the coexisting systems for every channel realisation. Both of these two categories of precoding approaches can avoid or control the CCI, therefore improving the system performance.

	Data CSI	Interference CSI	Data CSI (coexisting system)	Interference CSI (coexisting system)
MRT	√			
ZF		√		
IF	√	√		
IC	√	√		
Joint beamforming	√	√	√	√

Table 5.1: *Comparison of required CSI for linear precoding in MISO CR environments*

However, the cost of radio frequency (RF) chains is one of the main drawbacks of multiple antenna techniques. In a transmitter, a RF link usually includes low noise power amplifiers, gain control units, digital to analogue converters (DAC), and several filters. These components are the major cost of a transmitter. For each antenna, a RF link is required. Hence, increasing the number of antennas will lead to a significant increase in the cost of a transmitter. Another drawback of the beamforming for cognitive radio is the requirements for CSI. To control the CCI and optimise the performance, different CSI is required according to different precoding approaches. These requirements are listed in Table 5.1. It is clear that increasing the number of antennas needs a higher throughput feedback channel.

These two problems may be solved via antenna selection technologies. Antenna selection can reduce the hardware complexity whilst keeping much of the benefit of multiple antennas [66] [67]. The key idea behind this technology is using only a subset of available antennas to transmit or receive signals. For transmitter antenna selection, it means that we can select from many antennas with a few RF chains, therefore reducing the cost of the transmitters. The RF chains adaptively switch to a subset of all available antennas according to a specific optimisation criterion for a given channel realisation. In general, antenna selection has often been limited to receiver side since the transmitter antenna selection needs a feedback channel to transmit the CSI. However, it does not increase the complexity for a MISO cognitive radio system because the transmitters need to know the CSI to control the CCI in coexisting environments and feedback channel is necessary. Furthermore, if the receivers can process part of the antenna selection task, the capacity of the feedback channel can be decreased. For example, providing that the receivers decide which antennas are used in the transmitter, feedback of the CSI of all available antennas to the transmitters is not necessary. Only the selected CSI is needed; hence decreasing the data rate requirement of the feedback channel.

In this Chapter, we concentrate on the downlink of multiple input and single output cognitive radio (MISO-CR) systems with antenna selection technologies. Firstly, the scenario where only one antenna is selected in the transmitter side of the secondary system is considered. In this case, the primary system has to suffer interference from the secondary system since it is unavoidable when only one antenna is employed in the secondary system transmitter. Four different selection strategies, namely minimum interference, maximum data gain, maximum sum capacity, and the maximum signal to leak interference power ratio (SLIR) method proposed here, are analysed and discussed. The results show that our proposed SLIR approach achieves a better performance trade-off between two coexisting systems and improves the spectrum efficiency. Then we focus on the multiple antennas selection scenario where linear beamforming is used in the secondary system. We compare the optimal selection strategy, maximum norm, and subset optimal strategy in term of signal to noise power ratio (SNR). Our proposed strategy, the subset optimal approach, achieves near optimal performance and greatly reduces the computational complexity. These observations are confirmed by simulation results.

5.2 System model

Consider a typical point-to-point coexisting environment in which there are two independent radio systems using the overlapping spectrum, just as in Figure 5.1. For the sake of simplicity, we assume that the primary system (system A) is a single input and single output (SISO) radio system, and the signals of the primary system cannot be detected by the receiver of the secondary system, which could happen for example for an outdoor primary and indoor secondary system. However, the secondary system might interfere with primary system users' receivers. The channel of the primary system is defined as $h \in \mathbb{C}$. For the secondary system (system B), we assume that there are $N (N \geq 2)$ antennas but $n (n \leq N)$ RF chains in the transmitter side. The data channel and the interference channel for the secondary system are $\mathbf{h}_D = [h_{D1}, h_{D2}, \cdots, h_{DN}]$ and $\mathbf{h}_I = [h_{I1}, h_{I2}, \cdots, h_{IN}]$, where $h_{Dj}, h_{Ij} \in \mathbb{C}$. When transmitting signals, only n antennas are used among the N possible antennas. There are a total of $K = \binom{N}{n}$ possible antenna selections, and define the set of all possible selections as $\Omega = \{\omega_1, \omega_2, \cdots, \omega_K\}$. Without loss of generality, we define in each possible selection the

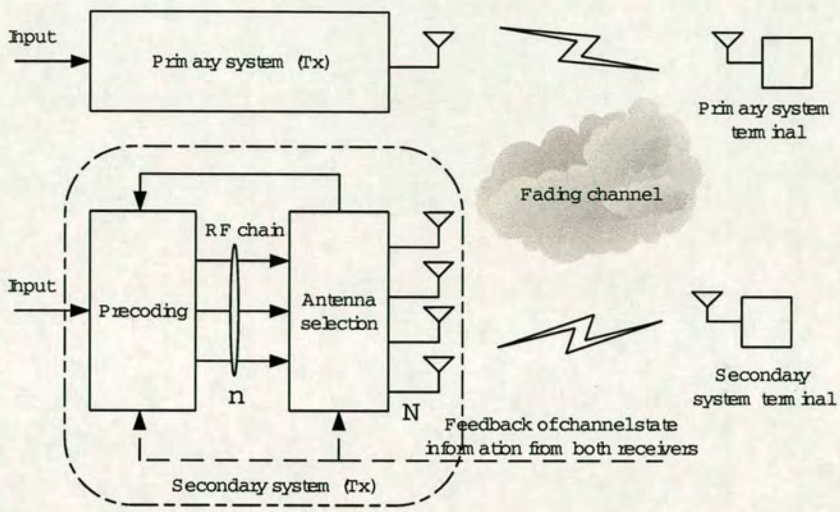


Figure 5.1: *Block diagram of MISO-CR system for antenna selection*

chosen antenna indices to be in increasing order, then

$$
\begin{aligned}
\omega_1 &= \{1, 2, \cdots, n\} \\
\omega_2 &= \{1, 2, \cdots, n-1, n+1\} \\
\omega_K &= \{N-n+1, \cdots, N\}.
\end{aligned}
$$

If the subset ω_j is selected, then channel is defined as $\mathbf{h}^j$.

The received signals y_A and y_B for the lth sample time can be written as

$$
\begin{aligned}
y_A(l) &= h(l)s_A(l) + r_{BA}\mathbf{h}_I^j(l)\mathbf{v}_B(l)s_B(l) + z_A(l) \\
y_B(l) &= \mathbf{h}_D^j(l)\mathbf{v}_B(l)s_B(l) + z_B(l)
\end{aligned}
$$

where s_A and s_B are the input signals for the primary system and the secondary system respectively, and the transmitter power constraint for system A and system B are denoted by $\mathrm{E}\left[\|s_A\|^2\right] \leq P_A$ and $\mathrm{E}\left[\|s_B\|^2\right] \leq P_B$. The vector $\mathbf{v}_B \in \mathbb{C}^{n\times 1}$ is the precoding vector for the secondary system providing that at least two antennas are selected, and we assume that $\|\mathbf{v}_B\|^2 = 1$. The additive noise terms z_A and z_B are independent complex Gaussian random variables with zero mean. Their variances are σ_A^2 and σ_B^2 respectively. The scalar r_{BA}, which is defined as the interference factor, can allow for interference reduction due to waveform design, frequency overlap, etc. Therefore, the received signal to interference and noise power ratio (SINR) of the primary system and the secondary system for antenna selection ω_j are:

$$\mathrm{SINR}_{\mathrm{A}}^{j} = \frac{P_A \left\| h \right\|^2}{\sigma_A^2 + r^2 P_B \|\mathbf{h}_I^j \mathbf{v}_B\|^2} \quad \mathrm{SINR}_{\mathrm{B}}^{j} = \frac{P_B \|\mathbf{h}_D^j \mathbf{v}_B\|^2}{\sigma_B^2}. \tag{5.1}$$

In the following discussion, we assume that the channel elements are independent, identically distributed (i.i.d) complex Gaussian random variables with zero mean and unit variance.

5.3 Single antenna selection

In this section we discuss the single-antenna selection strategies where $n = 1$. In such a scenario, the channel vectors which is selected for the secondary system simplify to scalars, and the precoding vector is 1. Furthermore, the primary system should have some kind of ability to tolerate the interference from the secondary system since it is normally unavoidable. However, single-antenna selection strategies are the cheapest solutions because only one RF chain is needed for the secondary system and usually the computational complexity of such strategies is linearly proportional to the number of available antennas N. We present four selection strategies, minimum interference, maximum data gain, maximum sum capacity, and maximum signal to leak interference power ratio (SLIR) in turn.

5.3.1 Minimum interference strategy

First, we consider the case where the primary system has less capability to tolerate CCI. In such an environment, the secondary system should ensure its interference is as small as possible. The obvious method is to select the antenna that has the minimum interference gain. It can be presented as follows:

$$\omega_{\min} = \arg\min_{\omega_j \in \Omega} \left\| \mathbf{h}_I^j \right\|^2 \tag{5.2}$$

The key point behind this idea is to exploit channel fading to reduce the interference. Normally, channel fading is a disadvantage of the radio system, and it sometimes may lead to low SNR in the receiver side because of the deep fading caused by multiple reflections. Providing that multiple antennas are used and the channels are independent, we have a higher probability to find a stronger channel in such system than in single antenna system therefore increasing the

SNR of the receiver. This is the so-called diversity gain. However, in the same situation, we also have a larger probability to find a weaker channel in MISO system than SISO system therefore decreasing the interference and increasing the SINR of the coexisting radio system. We call the decrease of interference caused by independence of fading channel as "interference diversity gain".

Lemma 5.1. *Let $\mathbf{h}_I$ be the interference channel whose elements are i.i.d complex Gaussian distributed with zero mean and unit variance, and interference channel gain $\zeta_j = \|h_{Ij}\|^2$ for $j \in \{1, 2, \cdots, N\}$. Define the random variable $\zeta_{\min} = \min \zeta_j$, $j \in \{1, 2, \cdots, N\}$; then the cumulative distribution function (CDF) $F_{\zeta_{\min}}(x)$ and probability density function (PDF) $f_{\zeta_{\min}}(x)$ of $\zeta_{\min}$ are:*

$$F_{\zeta_{\min}}(x) = 1 - e^{-Nx} \tag{5.3}$$

$$f_{\zeta_{\min}}(x) = Ne^{-Nx} \tag{5.4}$$

where $x \geq 0$; for $x < 0$, $F_{\zeta_{\min}}(x) = f_{\zeta_{\min}}(x) = 0$.

Proof. According to the definition in Lemma 5.1, we know that $\zeta_{\min} \geq 0$, then $F_{\zeta_{\min}}(x) = f_{\zeta_{\min}}(x) = 0$ for $x < 0$. Now we consider the case where $x \geq 0$. Since the elements of $\mathbf{h}_I$ are i.i.d complex Gaussian distributed with zero mean and unit variance, the variables ζ_k for any $k \in \{1, 2, \cdots, N\}$ are i.i.d random variables. Moreover for any $h_{Ik} = x_{Ik} + jy_{Ik}$, x_{Ik} and y_{Ik} are i.i.d random variables, and $x_{Ik}, y_{Ik} \sim N\left(0, \frac{1}{2}\right)$. Define $u_{Ik} = 2\zeta_{Ik} = \left(\sqrt{2}x_{Ik}\right)^2 + \left(\sqrt{2}y_{Ik}\right)^2$, so $u_{Ik} \sim \chi^2(2)$, and the PDF of u_{Ik} is $f_u(x) = \frac{1}{2}e^{-\frac{x}{2}}$ for $x \geq 0$. Therefore, the CDF of ζ_{Ik} for $x \geq 0$ is:

$$F_\zeta(x) = \Pr\{\zeta \leq x\} = \Pr\{u \leq 2x\} = \int_0^{2x} f_u(t)\,dt = 1 - e^{-x} \tag{5.5}$$

Moreover, the PDF ζ_{Ik} for $x \geq 0$ is

$$f_\zeta(x) = \frac{dF_\zeta(x)}{dx} = e^{-x} \tag{5.6}$$

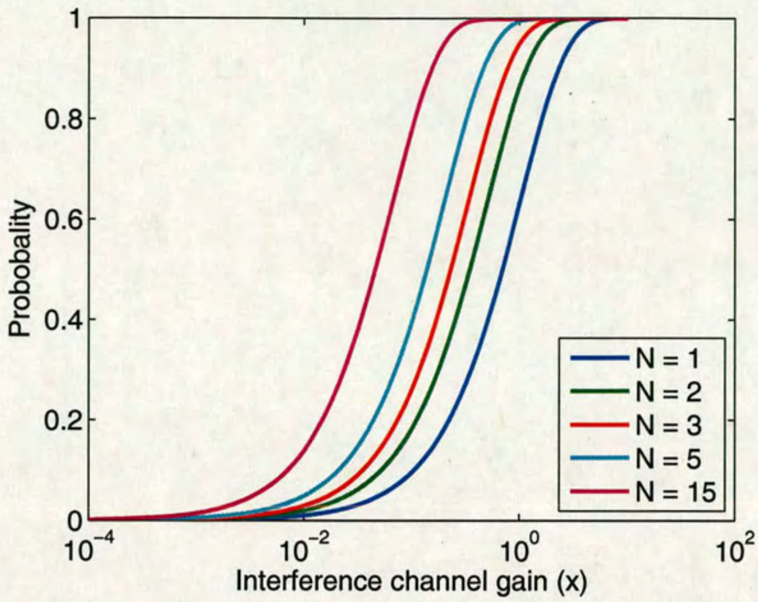


Figure 5.2: *Cumulative distribution function for* $\zeta_{\min}$

The CDF and PDF of $\zeta_{\min}$ for $x \geq 0$ are:

$$\begin{aligned} F_{\zeta_{\min}}(x) &= \Pr\{\zeta_{\min} \leq x\} = 1 - \Pr\{\zeta_{\min} > x\} = 1 - \bigcap_{k=1}^{N} \Pr\{\zeta_{Ik} > x\} \\ &= 1 - \prod_{k=1}^{N} [1 - \Pr\{\zeta_{Ik} \leq x\}] = 1 - [1 - F_{\zeta}(x)]^N = 1 - e^{-Nx} \end{aligned} \tag{5.7}$$

$$f_{\zeta_{\min}}(x) = \frac{dF_{\zeta_{\min}}(x)}{dx} = Ne^{-Nx} \tag{5.8}$$

□

The CDF and PDF of $\zeta_{\min}$ for different N are presented in Figure 5.2 and Figure 5.3. It is clear that when we increase the antenna array size N , the probabilities of obtaining a small interference gain increase.

The interference channel selection gain $\Phi_{\min}$ is the mathematical expectation of $\zeta_{\min}$

$$\Phi_{\min} = \mathrm{E}[\zeta_{\min}] = \int_0^{+\infty} x f_{\zeta_{\min}}(x) dx = \frac{1}{N} \tag{5.9}$$

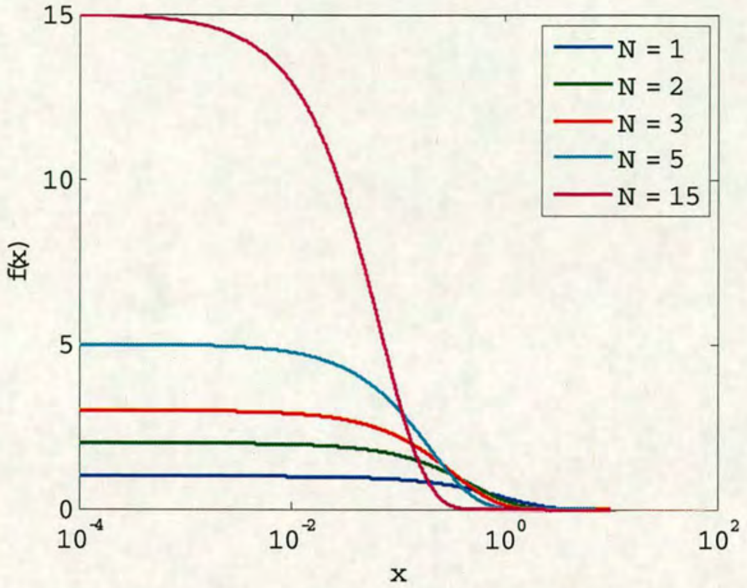


Figure 5.3: *Probability density function for* $\zeta_{\min}$

The trend of interference selection gain according to the available antenna number N is shown in Figure 5.4. It is clear that when N increases, the interference selection gain decreases, therefore the average interference to the primary system decreases. Doubling the number of available antennas will lead to 3dB decrease of interference. For an extreme situation, when $N \to \infty$, the interference selection gain $\Phi_{\min} \to 0$, which means no interference at all. On the other hand, given N antennas, adding one antenna can lead to a reduction of $\frac{1}{N(N+1)} \times 100\%$ percent of interference power compared to random selection. The interference decreases quickly when N is small; however, when N is large enough, the decrease of interference will be very small. For example, if $N = 10$, to add one more antenna will only reduce the interference by around 1%.

Theorem 5.2. *For the coexisting radio system defined in section 5.2, when the minimum interference power strategy of antenna selection is used for the secondary system, the CDFs of the SINR for both systems are:*

$$F_{\mathrm{SINR_A}}(x) = 1 - \frac{P_A N e^{-\frac{\sigma_A^2}{P_A}x}}{P_A N + r_{BA}^2 P_B x} \tag{5.10}$$

$$F_{\mathrm{SNR_B}}(x) = 1 - e^{\frac{-x\sigma_B^2}{P_B}} \tag{5.11}$$

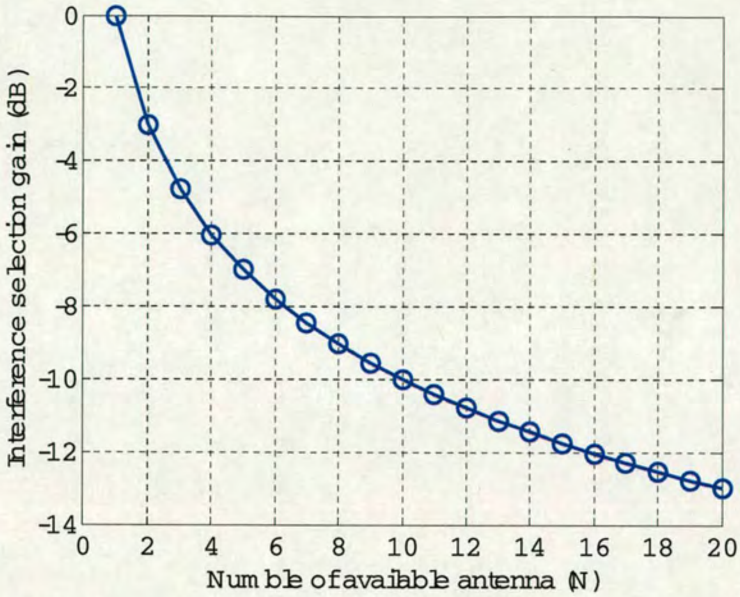


Figure 5.4: *Interference selection gain $\Phi_{\min}$ according to the available antenna N*

and the mathematical expectations for SINR$_{\mathrm{A}}$ and SINR$_{\mathrm{B}}$ are:

$$\mathrm{E}\left[\mathrm{SINR_A}\right] = \frac{NP_A}{r_{BA}^2 P_B} e^{\frac{N\sigma_A^2}{r_{BA}^2 P_B}} \mathrm{E}_1\left(\frac{N\sigma_A^2}{r_{BA}^2 P_B}\right) \tag{5.12}$$

$$\mathrm{E}\left[\mathrm{SNR_B}\right] = \frac{P_B}{\sigma_B^2} \tag{5.13}$$

where $\mathrm{E}_1(x)$ *is the order one exponential integral function [68] and*

$$\mathrm{E}_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt = -\gamma - \ln(x) - \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n!n} \qquad x \geq 0 \tag{5.14}$$

where γ *is the Euler constant, which is 0.5772156649.*

Proof. Since the SINRs are never less than zero, we only discuss the case where it greater or equal to zero. First, we consider the SNR performance of the secondary system. Since the interference channel and data channel are independent, the secondary system works like a one-antenna system when the minimum interference strategy is used. Assume the data channel

$h_B \in CN(0,1)$ then

$$F_{\mathrm{SNR}_B}(x) = \Pr\{\frac{P_B \|h_B\|^2}{\sigma_B^2} \leq x\} = \Pr\{\|h_B\|^2 \leq \frac{x\sigma_B^2}{P_B}\} = 1 - e^{-\frac{x\sigma_B^2}{P_B}} \tag{5.15}$$

$$\mathrm{E}[\mathrm{SNR}_B] = \mathrm{E}\left[\frac{P_B \|h_B\|^2}{\sigma_B^2}\right] = \frac{P_B}{\sigma_B^2} \tag{5.16}$$

where equation (5.15) follows from equation (5.5).

Define h and $h_{\min}$ to be the data channel of the primary system and the selected interference channel. Assume $q = \|h\|^2$ and $\zeta_{\min} = \|h_{\min}\|^2$, and according to our assumptions, these two random variables are independent. Therefore, we have

$$\begin{aligned} F_{\mathrm{SINR}_A}(x) &= \Pr\left\{\frac{P_A q}{r_{BA}^2 P_B \zeta_{\min} + \sigma_A^2} \leq x\right\} = \Pr\left\{q \leq \frac{x\left(r_{AB}^2 P_B \zeta_{\min} + \sigma_A^2\right)}{P_A}\right\} \\ &= \int_0^\infty \int_0^{\frac{x\left(r_{BA}^2 P_B y_2 + \sigma_A^2\right)}{P_A}} f_q(y_1)\, f_{\zeta_{\min}}(y_2)\, dy_1 dy_2 \end{aligned} \tag{5.17}$$

According to equation (5.6) and (5.8),

$$F_{\mathrm{SINR}_A}(x) = 1 - \frac{P_A N e^{-\frac{\sigma_A^2}{P_A}x}}{P_A N + r_{BA}^2 P_B x} \tag{5.18}$$

Moreover,

$$\mathrm{E}[\mathrm{SINR}_A] = \int_0^\infty \int_0^\infty \frac{P_A y_1}{r_{BA}^2 P_B y_2 + \sigma_A^2} f_q(y_1)\, f_{\zeta_{\min}}(y_2)\, dy_1 dy_2 \tag{5.19}$$

$$= N P_A \int_0^\infty \frac{e^{-N y_2}}{r_{BA}^2 P_B y_2 + \sigma_A^2} dy_2 \tag{5.20}$$

if $r_{BA}^2 P_B = 0$,

$$\mathrm{E}[\mathrm{SINR}_A] = \frac{P_A}{\sigma_A^2};$$

else define

$$t \triangleq N y_2 + u, \quad \text{where } u \triangleq \frac{N\sigma_A^2}{r_{BA}^2 P_B}.$$

So

$$\mathrm{E}[\mathrm{SINR}_A] = \frac{N P_A}{r_{BA}^2 P_B} e^u \int_u^\infty \frac{e^{-t}}{t} dt. \tag{5.21}$$

Since

$$\mathrm{E}_1(x) = \int_x^{\infty} \frac{e^{-t}}{t} dt = -\gamma - \ln(x) - \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n!n},$$

$$\mathrm{E}[\mathrm{SINR}_A] = \frac{NP_A}{r_{BA}^2 P_B} e^{\frac{N\sigma_A^2}{r_{BA}^2 P_B}} \mathrm{E}_1\left(\frac{N\sigma_A^2}{r_{BA}^2 P_B}\right).$$

□

Since the interference channel and the data channel for the primary system are independent, the antenna selection strategy according to the power of interference channel will not affect the average power for the data channel. Hence, for the data channel, the received power is equal to that for random selection and the antenna number N does not affect the performance of the secondary system, just like in equation (5.11).

Define $\phi(x) = xe^x \mathrm{E}_1(x)$, then equation (5.12) becomes

$$\mathrm{E}[\mathrm{SINR}_A] = \frac{P_A}{\sigma_A^2} \phi\left(\frac{N\sigma_A^2}{r_{BA}^2 P_B}\right) \tag{5.22}$$

So, the ergodic SINR of the primary system is proportional to $\phi(x)$. we also have that the signal-to-thermal noise ratio (ignoring interference) for the primary system, and it is:

$$\mathrm{E}[\mathrm{SNR}_A] = \frac{P_A \mathrm{E}\left[\|h\|^2\right]}{\sigma_A^2} = \frac{P_A}{\sigma_A^2}$$

then

$$\lim_{N\to\infty} \mathrm{E}[\mathrm{SINR}_A] = \mathrm{E}[\mathrm{SNR}_A] \lim_{u\to\infty} \phi(u) \;= \mathrm{E}[\mathrm{SNR}_A] \lim_{u\to\infty} \frac{\int_u^{\infty} \frac{e^{-t}}{t} dt}{\frac{e^{-u}}{u}} = \frac{P_A}{\sigma_A^2}$$

where

$$u = \frac{N\sigma_A^2}{r_{BA}^2 P_B}.$$

It is verified that if N is large enough or the interference power $r_{BA}^2 P_B$ small enough, the interference can be neglected. The curve of $\phi(x)$ is displayed in Figure 5.5. Here we give an example of the CDF of the SINR of the primary system. Define $P_A = P_B = 10\mathrm{dB}$, $\sigma_A^2 = \sigma_B^2 = 0\mathrm{dB}$, and $r_{BA} = 0.5$, then the CDF of the primary system SINR for $N = [1, 2, 3, 5, 20]$ antennas are shown in Figure 5.6. It is clear that increasing N leads to an improvement of the SINR performance for the primary system.

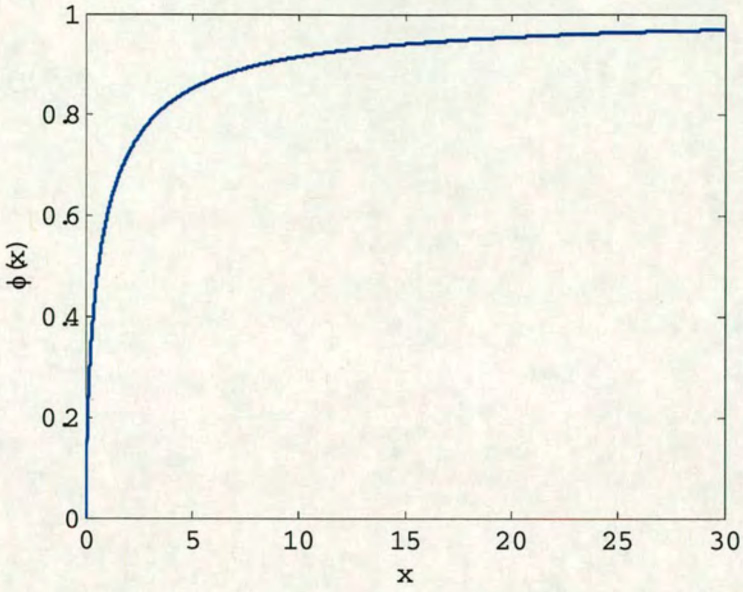


Figure 5.5: *The curve of $\phi(x)$*

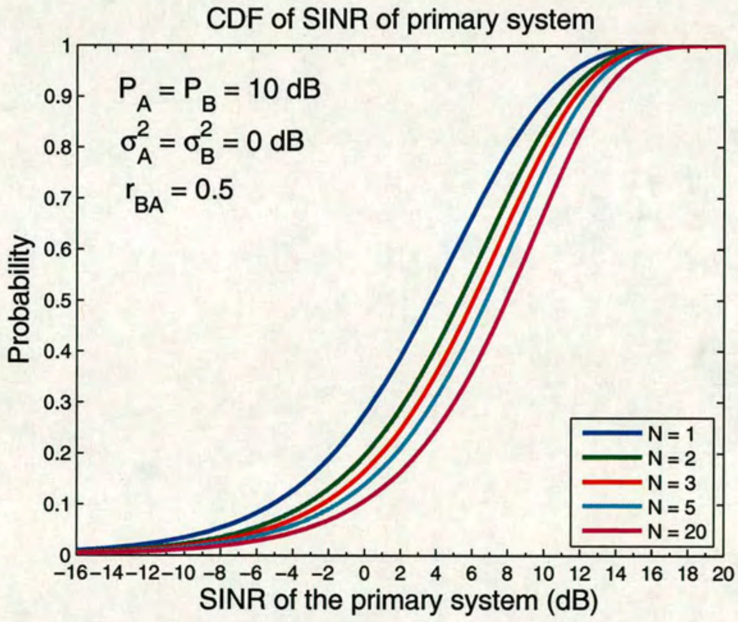


Figure 5.6: *CDF of SINR for primary system for minimum interference selection scheme*

To achieve good performance for the minimum interference strategy, the elements of the interference channel $\mathbf{h}_I$ should not correlate (the best performance is obtained when they are independent) and the number of available antennas N is large enough. The correlation between the elements of interference channel will degrade the performance of this method. However, we do not require the independence between the data channel elements. This strategy can be used in a scenario where the interference channel experiences significant multipath propagation. For example, a typical application for such method might be the coexistence between wireless local area networks (WLAN, such as 802.11g), which are usually an indoor networks, and outdoor WiMAX systems.

5.3.2 Maximum data channel power strategy

In the previous subsection, the case in which the secondary system tries to minimise the interference to the primary system is discussed. The minimum interference strategy is also equal to maximising the performance of the primary system in terms of SINR. However, the secondary system itself does not obtain any performance improvement through the antenna selection. Sometimes the primary system is not affected by the CCI caused by the secondary system due to the higher signal power or very small interference factor. In such situations, antenna selection is not used to reduce the interference, but to increase the performance of the secondary system. This idea has been illustrated in [69], and we apply it to our cognitive radio environment. We can select the antenna with the strongest channel gain, called the maximum data gain strategy, which is presented as follows:

$$\omega_{\max} = \arg \max_{\omega_j \in \Omega} \left\| \mathbf{h}_D^j \right\|^2 \tag{5.23}$$

Lemma 5.3. *Let $\mathbf{h}_D$ be the data channel whose elements are i.i.d complex Gaussian distributed with zero mean and unit covariance, and data channel gain for the secondary system $\eta_j = \|h_{Dj}\|^2$ for $j \in \{1, 2, \cdots, N\}$. Define the random variable $\eta_{\max} = \max \eta_j$, $j \in \{1, 2, \cdots, N\}$; then the cumulative distribution function $F_{\eta_{\max}}(x)$ and probability density $f_{\eta_{\max}}$ function of $\eta_{\max}$ are:*

$$F_{\eta_{\max}}(x) = \left(1 - e^{-x}\right)^N \tag{5.24}$$

$$f_{\eta_{\max}}(x) = N \left(1 - e^{-x}\right)^{N-1} e^{-x} \tag{5.25}$$

where $x \geq 0$. For $x < 0$, $F_{\eta_{\max}}(x) = f_{\eta_{\max}}(x) = 0$.

Proof. It has been shown in [69] (pp.337) □

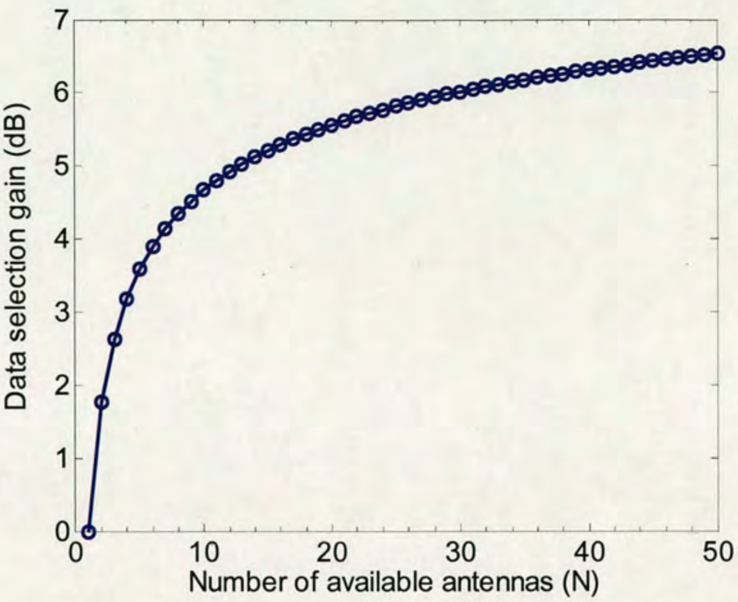


Figure 5.7: *Data selection gain* $\Psi_{\max}$

Then the data channel selection gain $\Psi_{\max}$ is [69] (Appendix VI):

$$\Psi_{\max} = \int_0^{\infty} xN\left(1-e^{-x}\right)^{N-1} e^{-x} dx = \sum_{k=1}^{N} \frac{1}{k} \tag{5.26}$$

From equation (5.26), we can obtain an additional gain of $\frac{1}{N+1}$ compared to the random selection strategy when we increase the number of antenna from value N to $N+1$. The larger the N, the less the benefit we obtain from adding one more antenna. If we let $N \to \infty$, we have $\Psi_{\max} \to \infty$ because the harmonic series is an unbounded series [68]. It means that we can infinitely increase the selection gain, and hence the performance of the secondary system, via increasing the number of available antennas. The curve of $\Psi_{\max}$ to the number of antennas N is shown in Figure 5.7.

Now we discuss the SINR performance of both systems. First consider the primary system: since the interference channels and data channels are independent and the elements of the interference channel are i.i.d random variables, selecting the antenna with maximum data channel gain has no influence on the interference channel. So for the interference channel, it is equal to random selection, which has same effect as $N = 1$. Then the CDF of SINR and ergodic SINR for the primary system can be obtained from equation (5.10) and (5.12):

$$F_{\mathrm{SINR}_A}(x) = 1 - \frac{P_A e^{-\frac{\sigma_A^2}{P_A}x}}{P_A + r_{BA}^2 P_B x} \tag{5.27}$$

$$\mathrm{E}\left[\mathrm{SINR}_A\right] = \frac{P_A}{r_{BA}^2 P_B} e^{\frac{\sigma_A^2}{r_{BA}^2 P_B}} \mathrm{E}_1\left(\frac{\sigma_A^2}{r_{BA}^2 P_B}\right) \tag{5.28}$$

Only decreasing the transmitter power of the secondary system can improve the SINR performance of the primary system. Then consider the secondary system, the CDF of SNR is

$$\begin{aligned} F_{\mathrm{SNR}_B}(x) &= \Pr\left\{\frac{P_B \eta_{\max}}{\sigma_B^2} \le x | x \ge 0\right\} = \Pr\left\{\eta_{\max} \le \frac{x\sigma_B^2}{P_B}\right\} \\ &= \left(1 - e^{\frac{x\sigma_B^2}{P_B}}\right)^N \end{aligned} \tag{5.29}$$

and the ergodic SNR is

$$\mathrm{E}\left[\mathrm{SNR}_B\right] = \mathrm{E}\left[\frac{P_B \eta_{\max}}{\sigma_B^2}\right] = \frac{P_B}{\sigma_B^2} \sum_{k=1}^{N} \frac{1}{k} \tag{5.30}$$

As in the minimum interference strategy, the maximum data power strategy only needs the independence of the data channel elements. The dependency between the elements of the interference channel will not influence the primary system performance.

5.3.3 Maximum sum capacity strategy

We considered two kinds of extreme situations in previous subsections: one is the best for the primary system and the other for the secondary system. However, here our aim is to achieve the maximum spectrum efficiency and find the best trade-off between two coexisting radio systems. Therefore, the maximum sum throughput strategy, which selects the antenna that can achieve optimal sum capacity, can be used.

$$\omega_{\mathrm{max-c}} = \max_{\omega_j \in \Omega} \left[\log\left(1 + \mathrm{SINR}_A^j\right) + \log\left(1 + \mathrm{SNR}_B^j\right)\right] \tag{5.31}$$

From equation (5.31), it is clear that to select the optimal antenna from N available antennas, the transmitter of the secondary system needs to compute all possible SINRs for both systems.

This means that all the CSI should be known by the transmitter of the secondary system, including the CSI of the primary system. Moreover, the primary system and the secondary system have equal priorities. The channel conditions, transmitter power, and noise in the receiver decide which system has better performance. However, in fact, the secondary system has a high probability to achieve better performance because there is no co-channel interference (CCI) for the secondary system in the CR model of this chapter while the primary system usually has to suffer serious CCI.

5.3.4 Maximum signal to leak interference power strategy

Three selection strategies have been discussed in the previous subsections. In the minimum interference strategy, although we ensure the primary system experiences less interference, the secondary system has not obtained any benefit from the use of multiple antennas. If we allow more interference to the primary system, it is possible that the performance of the secondary system could be greatly improved. For the maximum data gain strategy, it may greatly decrease the performance of the primary system, and this is usually not desirable in a cognitive radio environment. Moreover, although the maximum sum capacity strategy can achieve best overall performance, it might sometimes greatly degrade the performance of the primary system, just like maximum data strategy. So we need to find an antenna selection strategy that has little negative influence on the primary system while the secondary system may benefit from it, and then increase the total spectrum efficiency.

We propose a novel maximum signal to leak interference ratio (SLIR) strategy to solve this problem. The key idea is to select the antenna with maximum SLIR. Define the SLIR for antenna j as

$$Q_j = \frac{\|h_{Dj}\|^2}{\|h_{Ij}\|^2}, \tag{5.32}$$

where $j \in \{1, 2, \cdots, N\}$. Then the maximum SLIR strategy can be presented as follows:

$$\omega_{\max-\mathrm{SLIR}} = \arg \max_{\omega_j \in \Omega} Q_j. \tag{5.33}$$

From the definition of SLIR, it is easy to understand that the interference channel and data channel are both considered in this method, and the interference channel gain is more important than data channel gain. It has a higher probability to select an antenna which causes lower interference than an antenna which has high data channel gain. This can ensure that the performance of the primary system has little negative influence. On the other hand, this method does also consider the data channel. If two antennas leak similar levels of interference power, the antenna with the higher data channel gain will be selected, and the performance of the secondary system is improved. In fact, this strategy allows a trade-off in performance between the two coexisting radio systems.

Lemma 5.4. *With the assumptions in section 5.2, the random variables Q_j for $j \in \{1, 2, \cdots, N\}$ are i.i.d; the CDF and PDF of random variables Q_j are*

$$F_Q(x) = \frac{x}{x+1} \tag{5.34}$$

$$f_Q(x) = \frac{1}{(x+1)^2} \tag{5.35}$$

where $x \geq 0$. For $x < 0$, $F_Q(x) = f_Q(x) = 0$.

Proof. Define h_D and h_I to be the data and interference channels for one arbitrarily chosen antenna. Assume that $\eta_D = \|h_D\|^2$ and $\zeta_I = \|h_I\|^2$, and these two random variables are independent and have same probability density function $f(x)$. Therefore, when $x \geq 0$,

$$\begin{aligned} F_Q(x) &= \Pr\{Q \leq x\} = \Pr\{\eta_D \leq x\zeta_I\} \\ &= \int_0^\infty \int_0^{xy_2} f(y_1) f(y_2)\, dy_1 dy_2 = \frac{x}{x+1} \end{aligned} \tag{5.36}$$

and

$$f_Q(x) = \frac{dF_Q(x)}{dx} = \frac{1}{(x+1)^2} \tag{5.37}$$

□

Since the Q_j are i.i.d random variables, then CDF of $Q_{\max}$ is

$$F_{Q_{\max}}(x) = \prod_{j=1}^{N} \Pr\{Q_j \leq x\} = \left(\frac{x}{x+1}\right)^N \tag{5.38}$$

the PDF of the $Q_{\max}$ is

$$f_{Q_{\max}}(x) = \frac{dF_{Q_{\max}}(x)}{dx} = N\frac{x^{N-1}}{(x+1)^{N+1}}. \tag{5.39}$$

Unlike the minimum interference and maximum data gain strategies, this method needs to compute the maximum SLIR at the transmitter side. Therefore, the receivers have to feed back the CSI of all available antennas for the secondary system.

5.4 Selection of multiple antennas

Single antenna selection strategies have been discussed in the previous section but their major weakness is the interference to the primary system. Although in theory, single antenna selection can fully cancel the interference when N tends to infinity, it is not a practical method. Normally, to keep the interference below an acceptable value, we need a larger number of antennas. However, if we do not just select one antenna, but select multiple antennas, we can use the beamforming technologies described in previous chapters to cancel or control the interference to the primary system. The cost of multiple-antenna selection is the complexity of the selection strategy, more CSI, and increased RF costs compared to single antenna selection schemes.

In this section, three multiple-antenna selection strategies, including optimal, maximum norm, and our proposed subset optimal strategy are discussed. The received SNR of the secondary system is our criterion for optimisation whilst satisfying the interference requirements of the primary system. Linear precoding methods, IF and IC are used for the secondary system. The IF and IC methods achieve optimal linear beamforming performance when no interference and controlled interference to the primary system is allowed. The precoding vectors for IF and IC are (Chapter 3) [62]:

$$\mathbf{v}_B^{IF} = \left[\mathbf{I}_n - \frac{\left(\mathbf{h}_I^j\right)^H \mathbf{h}_I^j}{\|\mathbf{h}_I^j\|^2} \right] \frac{\left(\mathbf{h}_D^j\right)^H}{\left\|\mathbf{h}_D^j\right\| \sqrt{1-\rho^2}}$$

$$\mathbf{v}_B^{IC} = \sqrt{1-\beta^2}\mathbf{v}_B^{IF} + \beta \frac{\mathbf{h}_D^j \left(\mathbf{h}_I^j\right)^H}{\rho} \left(\mathbf{h}_I^j\right)^H$$

and

$$\beta = \min\left(\rho, \sqrt{\frac{J}{r_{BA}^2 P_B \|\mathbf{h}_I^j\|^2}}\right)$$

where $\min(x, y)$ is the minimum value of x and y, and J is interference constraint to the primary system. Their corresponding channel gains are (Chapter 3) [62]:

$$\overline{G}_{IF} = \left(1-\rho^2\right) \|\mathbf{h}_D^j\|^2 \tag{5.40}$$

$$\overline{G}_{IC} = \left(\sqrt{\left(1-\beta^2\right)\left(1-\rho^2\right)} + \beta\rho\right)^2 \left\|\mathbf{h}_D^j\right\|^2 \tag{5.41}$$

where ρ is the correlation coefficient between the data and interference channel of the secondary system and β is the interference coefficient chosen to obey the IC interference constraint. For a given β, the interference power to the primary system is $J = \beta^2 \|\mathbf{h}_I^j\|^2 P_B r_{BA}^2$. For the sake of the simplicity, we unify the equation (5.40) and (5.41) to one equation as follows:

$$\overline{G} = \vartheta \left\|\mathbf{h}_D^j\right\|^2 \tag{5.42}$$

where the value of ϑ depends on which beamforming method is used and is a function of β and ρ . Moreover, $\vartheta \in [0, 1]$. The interference gain is defined as $\overline{I} = \beta^2 \|\mathbf{h}_I^j\|^2$.

5.4.1 Optimal strategy

Firstly, we consider the multiple antenna selection strategy that can achieve the optimal SNR performance for the secondary system. The basic idea of this method is to explore all the possible combinations of antennas and select a combination which can maximise the data channel gain and therefore maximise the SNR of the secondary system subject to the interference constraint. This method can be expressed as:

$$\omega_{op} = \arg\max_{\omega_j \in \Omega} \overline{G}. \tag{5.43}$$

As shown in equation (5.40), (5.41), and (5.42), the value of $\overline{G}$ not only relates to the norm of the selected channel, but also the correlation coefficient of the interference and the data channels ρ and/or interference coefficient β. Although this method can achieve the best performance, it has a high computational complexity. To select n out of N antennas with an optimal strategy, we need to compute $K = \binom{N}{n} = \frac{N!}{n!(N-n)!}$ values of $\overline{G}$, whose size increases very quickly when N increases. For example, if we select 4 antennas from 10, we need to compare 210 values; if we add two more available antennas (select 4 antennas from 12), it more than doubles to 495 values. Another drawback of the optimal strategy is the requirement for CSI. Since all the combinations need to be estimated, both the interference channel and data channel state information for all available transmit antennas is required.

The performance of the optimal strategy normally is difficult to obtain since the set of random variables of channel gain $\overline{G}$ for all antenna combinations are not mutually independent of each other. Only single antenna selection can ensure the independence of channel gain. For multiple antenna selection, each antenna will be included in several different combinations, therefore leading to correlation between these combinations. Moreover, the common antennas for two arbitrary combinations are different and the number of common antennas may range from $\max(2n - N, 0)$ to $n - 1$ when $n > 1$. This leads to the different correlation values between the antenna subsets and therefore increases the difficulty of performance estimation of the optimal strategy.

Theorem 5.5. *Consider the scenario described in Section 5.2. Select n antennas from N available antennas for the secondary system while IF is used. If the data channel elements and interference channel elements are i.i.d complex Gaussian random variables with zero mean and unit variance, the cumulative distribution function (CDF) of $\overline{G}_{IF}^{op}$ satisfies:*

$$F(x) \geq F_{\overline{G}_{IF}^{op}}(x) \geq F_1(x) \tag{5.44}$$

and

$$\begin{aligned} F_1(x) &= KF(x) + 1 - K & (5.45) \\ F(x) &= \frac{\gamma(n-1, x)}{(n-2)!} & (5.46) \end{aligned}$$

where $\gamma(a, x)$ is lower incomplete gamma function [68], defined as

$$\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt$$

and

$$K = \frac{N!}{n!(N-n)!}$$

Proof. Consider any selected combination ω_j, then rewrite the equation (5.40) as

$$\overline{G}_{IF}^{j} = \left\| \mathbf{h}_D^j \right\|^2 - \left\| \sum_{k \in \omega_j} \frac{h_{Ik}}{\|\mathbf{h}_I^j\|} h_{Dk}^* \right\|^2 . \tag{5.47}$$

Define a $n \times n$ unitary matrix $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_n]$, and

$$\mathbf{a}_1 = \left(\frac{\mathbf{h}_I^j}{\left\| \mathbf{h}_I^j \right\|} \right)^T.$$

Further define the size n random vector $\mathbf{u} = [u_1, u_2, \cdots, u_n]^T$, and $\mathbf{u}$ is the linear transformation of $\left(\mathbf{h}_D^j\right)^H$ via $\mathbf{A}$. It means that

$$\mathbf{u} = \mathbf{A}\left(\mathbf{h}_D^j\right)^H \quad \text{and} \quad u_1 = \sum_{k \in \omega_j} \frac{h_{Ik}}{\left\| \mathbf{h}_I^j \right\|} h_{Dk}^*. \tag{5.48}$$

Since $\mathbf{A}$ is a unitary matrix, and the elements of $\mathbf{h}_D^j$ are i.i.d complex Gaussian random variables with zero mean and unit variance, $u_1, u_2, \cdots, u_n$ also are i.i.d complex Gaussian random variables with zero mean and unit variance [70]. Furthermore,

$$\|\mathbf{u}\|^2 = \mathbf{h}_D^j \mathbf{A}^H \mathbf{A}\left(\mathbf{h}_D^j\right)^H = \left\| \mathbf{h}_D^j \right\|^2. \tag{5.49}$$

Then equation (5.47) becomes:

$$\overline{G}_{IF}^j = \|\mathbf{u}\|^2 - \|u_1\|^2 = \sum_{k=2}^{n} \|u_k\|^2. \tag{5.50}$$

Thus, $2\overline{G}_{IF}^j \sim \chi^2(2n-2)$. So the CDF of channel gain for antenna combination ω_j is [71]:

$$F_{\overline{G}_{IF}^j}(x) = \Pr\left\{2\overline{G}_{IF}^j \leq 2x\right\} = \frac{\gamma(n-1, x)}{(n-2)!} \tag{5.51}$$

where

$$\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt$$

is the lower incomplete gamma function [68]. Therefore, channel gains for any antenna combination are identically distributed random variables and define their CDFs as $F(x) = F_{\overline{G}_{IF}^j}(x)$.

Furthermore, the CDF of optimal antenna combination which maximises the data channel gain can be written as:

$$F_{\overline{G}_{IF}^{op}}(x) = \Pr\left\{ \max_{\omega_j \in \Omega} \left\{ \overline{G}_{IF}^j \right\} \leq x \right\} \tag{5.52}$$

Since the random variables sequence G_{IF}^{j} are not fully independent, then [72]

$$F(x) \geq F_{\overline{G_{IF}^{op}}}(x) \geq F_1(x) \tag{5.53}$$

where

$$\begin{aligned} F_1(x) &= KF(x) + 1 - K && (5.54)\\ K &= \frac{N!}{n!(N-n)!} && (5.55) \end{aligned}$$

□

Theorem 5.5 supplies a lower bound and a upper bound of the CDF of the data channel gain for the optimal strategy when IF is used. However, the lower bound is not tight since only when $F(x) \geq 1 - \frac{1}{K}$, $F_1(x) \geq 0$. If a constraint is given to n, a tighter lower bound can be obtained.

Corollary 5.6. *If $n = N - 1$, then the cumulative distribution function (CDF) of $\overline{G_{IF}^{op}}$ satisfies:*

$$F_{\overline{G_{IF}^{op}}}(x) > \left[\frac{\gamma\,(n-1,x)}{(n-2)!}\right]^{n+1}. \tag{5.56}$$

And it is a tighter lower bound compared to Theorem 5.5.

Proof. Since $n = N - 1$, then $K = \binom{N}{n} = N = n + 1$. For each antenna combinations, there is only one antenna which has not been included. Moreover, for any two combinations, there are $n - 1$ common antennas and one different antenna. Since the channels are i.i.d random variables, we can say that the random variables sequence of data channel gains of IF for each antenna combination are exchangeable. Therefore, according to equation (5.52) and [73], the cumulative distribution function (CDF) of $\overline{G_{IF}^{op}}$ satisfies:

$$F_{\overline{G_{IF}^{op}}}(x) > F^N(x) = \left[\frac{\gamma\,(n-1,x)}{(n-2)!}\right]^{n+1}. \tag{5.57}$$

Now let us prove it is a tighter lower bound compared to Theorem 5.5. Define

$$g(x) = F^K(x) - F_1(x) \tag{5.58}$$

then

$$\begin{aligned} g(x)|_{K=N} &= F^N(x) - [1 - N + NF(x)] \\ &= [1 - F(x)]\,[N - \sum_{k=1}^{N} F^{N-k}(x)] \end{aligned}$$

Since $F(x)$ is the CDF of the random variable, $F(x) \leq 1$. Then

$$N - \sum_{k=1}^{N} F^{N-k}(x) \geq 0.$$

Therefore,

$$g(x)|_{K=N} \geq 0. \tag{5.59}$$

So $F^K(x) \geq F_1(x)$ and $F^K(x)$ is a tighter lower bound compared to $F_1(x)$. □

5.4.2 Maximum norm strategy

The optimal strategy can achieve best performance but it has a high computational complexity and requires full CSI. This is because it needs to consider the correlation and the amplitude at same time. If we just consider one part of the channel gain, the computational complexity may be decreased. Just to consider the correlation is not a good idea. From equation (5.40) and (5.41), it is easy to see that the correlation ρ between selected interference channel and data channel should tend to zero if we would like to maximise the channel gain without considering the norm $\left\|\mathbf{h}_D^j\right\|$. However, to perform this process, we need to calculate the correlation for each possible combination and full CSI is required for the transmitter. Therefore, it has the same complexity as the optimal method, but obtains worse results. The other part is the norm of the selected data channel. To achieve the best performance without the correlation value we need to maximise the norm $\left\|\mathbf{h}_D^j\right\|$. Selecting the antennas with the largest n amplitudes can ensure the largest norm. Moreover, this calculation can be performed at the receiver side and it only needs to feed back selected antenna indices and corresponding channel information. Therefore the maximum norm strategy not only reduces the computational complexity significantly but also decreases the bandwidth requirements of the feedback channel. The maximum norm strategy is presented as:

$$\omega_{\text{max-norm}} = \arg\max_{\omega_j \in \Omega} \left\|\mathbf{h}_D^j\right\|^2. \tag{5.60}$$

For a given channel, define $h_D^{(1)}, h_D^{(2)}, \cdots, h_D^{(N)}$ as the ordered data channel elements, where $h_D^{(1)}$ has the largest amplitude and $h_D^{(N)}$ is the smallest one. Then the channel gain with the maximum norm strategy should be

$$\overline{G} = \vartheta \sum_{k=1}^{n} \left\| h_D^{(k)} \right\|^2. \tag{5.61}$$

Since only the norm part is considered, the SNR performance of the secondary system will be poorer than the optimal strategy. Now we discuss the average SNR received by the secondary system for IF precoding. Define θ be the angle between the interference channel and the data channel, and $\sin^2\theta = 1 - \rho^2$, where $\theta \in [0, \pi/2]$; and the maximum norm is $u = \sum\limits_{k=1}^{n} \left\| h_D^{(k)} \right\|^2$. Because the angle and the amplitude are independent, the random variables θ and u are also independent random variables. Thus, from equation (5.1) and (5.40), the average SNR for the secondary system is

$$\mathrm{E}\left[\mathrm{SNR}_B\right] = \frac{P_B}{\sigma_B^2} \mathrm{E}\left[\sin^2\theta\right] \mathrm{E}\left[u\right] \tag{5.62}$$

From [74], the probability density function (PDF) of θ is

$$f_\theta(x) = 2(n-1)\sin^{2n-3} x \cos x,$$

where $x \in [0, \pi/2]$. Then

$$\mathrm{E}\left[\sin^2\theta\right] = \int_0^{\frac{\pi}{2}} \sin^2 x f_\theta(x)\, dx = 1 - \frac{1}{n}. \tag{5.63}$$

And according to [75],

$$\mathrm{E}\left[u\right] = n + \sum_{k=n+1}^{N} \frac{n}{k}.$$

Therefore, average SNR of the secondary system can be obtained from equation (5.63) and (5.62)

$$\mathrm{E}\left[\mathrm{SNR}_B\right] = \frac{P_B}{\sigma_B^2} \left[n - 1 + (n-1) \sum_{k=n+1}^{N} \frac{1}{k} \right] \tag{5.64}$$

where $n \geq 2$. By adding one more available antenna to N antennas, the SNR will increase by a factor $\frac{n-1}{N+1} \times 100\%$.

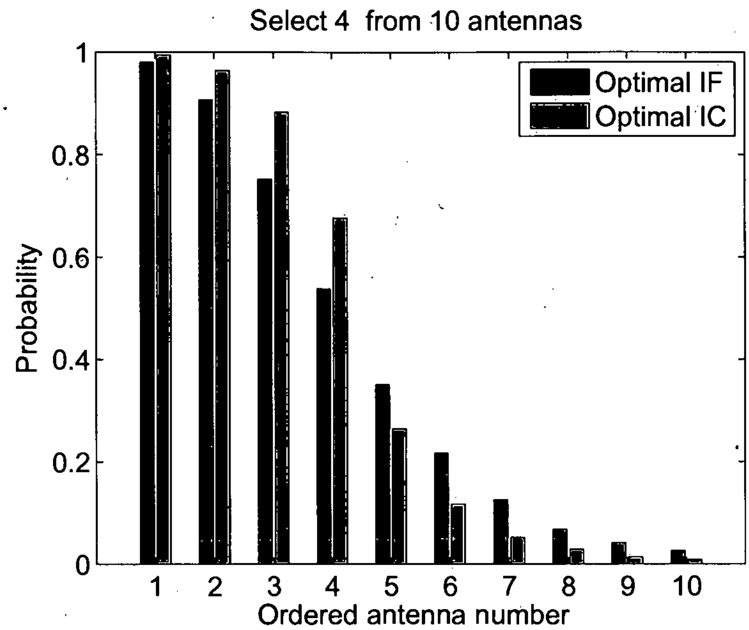


Figure 5.8: *Antenna selection probability with ordered channel gain for optimal strategy*

5.4.3 Subset optimal strategy

We discussed the optimal strategy and the maximum norm strategy in previous subsections. The optimal strategy is not a practical method due to the computational complexity. Although the maximum norm method is easy to implement, it will lose some performance. The antenna selection probability according to amplitude for optimal selection strategy is shown in Figure 5.8, where the power of additive white noise for both systems are 0dB and interference power constraint to the primary system $J \leq 0.2$ for IC. In this figure, we order antennas by decreasing norm, then plot the probabilities that they are selected by the optimal antenna selection method. The scenario is to select the best 4 antennas from 10 antennas with IF and IC precoding. It is clear that although the largest 4 antennas have a larger selection probabilities, the antennas with smaller norm sometimes are still selected. This difference of selection probabilities between the optimal strategy and the maximum norm strategy can explain why performance is lost for maximum norm strategy.

We will now propose a novel method to improve performance compared to the maximum norm approach. Reconsider the optimal method. The reason for the high computational complexity is exploring all the possible combinations. If we decrease the number of elements, the computational complexity will definitely decrease. According to this analysis, we propose a subset

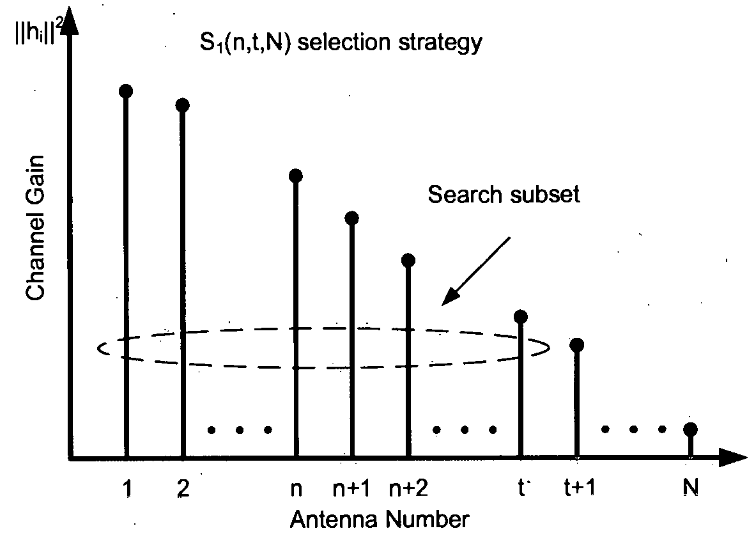


Figure 5.9: *Subset optimal antenna selection strategies* $S_1(n,t,N)$

optimal antenna selection strategy. This method has two steps:

1. Select a subset from all the available antennas according to a certain rule (maximum norm is used here);

2. Explore all possible antenna combinations among the selected subset to find the optimal solution;

The key point for this method is how to select the subset. From Figure 5.8, we can see that the larger the amplitude, the higher probability to be selected. Although sometimes antennas with the smaller amplitude are selected, the selection probabilities are very small. So if we do not consider these smallest-amplitude antennas, we should not lose a lot of performance. Therefore, we can use the amplitude as our subset selection criterion. This method is explained in Figure 5.9. Define the subset selection strategy $S_1(n,t,N)$, where N is the number of available antennas, t $(N \geq t \geq n)$ is the size of the subset selected from the N available antennas (shown by an ellipse in Figure 5.9) and this subset includes the t largest amplitude antennas; n is the number of selected antennas, which is equal to the number of RF chains.

The computational complexity for $S_1(n,t,N)$ is $K_1 = \binom{t}{n}$. Increasing the size of subset t will increase the computational complexity. Since the process of selecting the subset can be

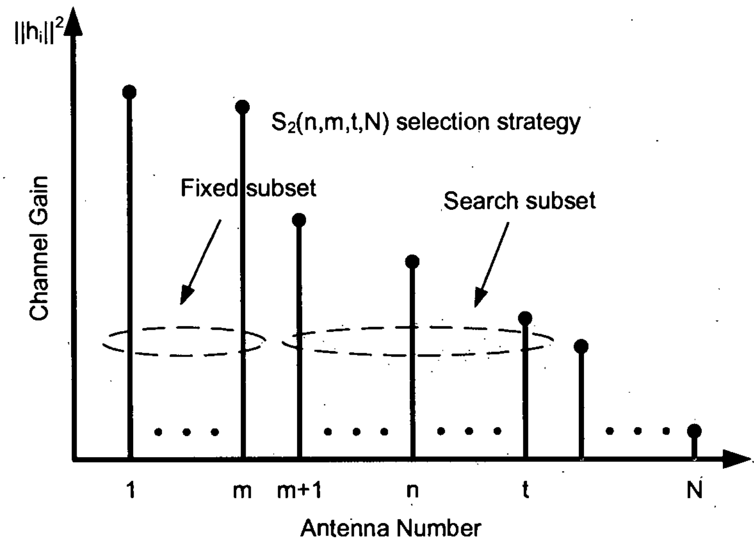


Figure 5.10: *Subset optimal antenna selection strategies* $S_2(n, m, t, N)$

performed in the receiver side, t also specifies the amount of feedback information. The larger value of t, the more computation is required and the more feedback information is needed, but the better the performance is achieved. If $t = N$, this method becomes the most complex optimal strategy; however if $t = n$, this method becomes the maximum norm strategy.

To further decrease the computational complexity, we can use the subset optimal selection strategy $S_2(n, m, t, N)$, shown in Figure 5.10. We still select a subset from all available antennas, and the definitions of N, t, and n are the same as $S_1(n, t, N)$. The key idea for this strategy is that the largest m $(0 \leq m \leq n)$ antennas are fixed and definitely selected (the left ellipse in Figure 5.10) so we do not need fully explore the selected subset. We will not lose much performance because the probabilities of the antennas with largest amplitude tend to one, just as shown in Figure 5.8. Then the size of the subset we need to explore is $(t - m)$. Thus, the computational complexity for $S_2(n, m, t, N)$ is $K_2 = \binom{t-m}{n-m}$. Increasing t or decreasing m will lead to an improvement of performance at the cost of higher computational complexity. Another thing need to be noted is that the required feedback information of $S_1(n, t, N)$ and $S_2(n, m, t, N)$ are the same, and both of them are related to the subset size t.

5.5 Simulations

In this section, simulations are used to verify our analysis. For the single-antenna selection strategies, the capacity performance is estimated and shown in subsection 5.5.1. The simulation results of multiple-antenna selection are shown in subsection 5.5.2. We use the SNR to compare the performance for difference selection strategies. Furthermore, the computational complexity and required CSI are displayed. We use the i.i.d Rayleigh fading channel in all these simulations, and the noise power for both coexisting systems are $\sigma_A^2 = \sigma_B^2 = 1.0$ (0dB). In the following discussion, the transmission power is defined as a relative value (dB) and the reference value is the noise power.

5.5.1 Single antenna selection

In this subsection, we compare the capacity performance of the four single-antenna selection strategies described in Section 5.3. Assume that the transmitter power of the primary system P_A is 10dB, and the number of available antennas for the secondary system N is 4. Define the interference factor r_{BA} to be 0.5, and we assume there is no interference from the primary system to the secondary system. Then we compare the capacity performance of minimum interference strategy, maximum data power strategy, maximum sum capacity strategy, and maximum signal to leak interference ratio strategy.

The capacities of the primary system are displayed in Figure 5.11. It is clear that the minimum interference method achieves the best capacity performance. It is because for this strategy, space diversity is fully used to reduce the co-channel interference. As we expect, the maximum data power method obtains the worst performance for the primary system since it does not consider the CCI. For our proposed maximum SLIR strategy, it just loses a little capacity performance compared to the minimum interference strategy. For example, when the transmitter power of the secondary system $P_B = 8$dB , only 0.11 bits/s/Hz are lost compared to the minimum interference strategy. However, the maximum data power strategy loses 0.55 bits/s/Hz compared to the minimum interference strategy at the same transmitter power.

The capacity performance of the secondary system is shown in Figure 5.12. Here, the maximum data power method obtains the best performance and the minimum interference method achieves the worst performance. The performance of our proposed method is better than the minimum interference method and worse than the maximum data methods. We still look at

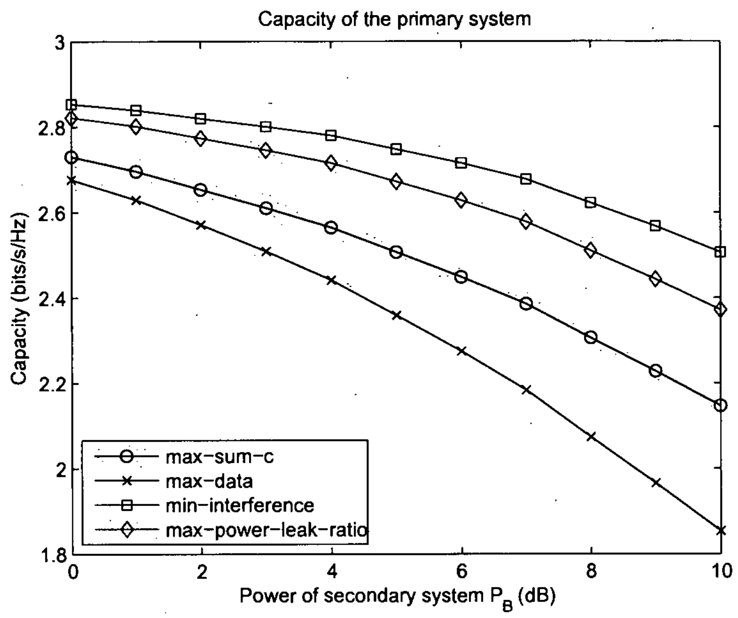


Figure 5.11: *Capacity of the primary system for single antenna selection schemes*

the case where P_B = 8dB, the maximum SLIR method outperforms by 0.82 bits/s/Hz the minimum interference method but loses 0.57 bits/s/Hz compared to the maximum data power method.

Finally, we compare the sum capacities that are presented in Figure 5.13. Although the minimum interference method is the safest method, it will lose some sum capacity performance. The maximum data method achieves near optimal sum capacity because it greatly improves the performance of the secondary system and the interference factor is not large and the transmission power of the secondary system is small. All of these lead to a small interference to the primary system although maximum data method is used. However, when the transmission power of the secondary system increases, the sum capacity performance of maximum data scheme will degrade quickly. Our proposed method improves the sum capacity compared to the minimum interference method whilst having little influence on the primary system. For P_B = 8dB, it loses 0.26 bits/s/Hz for sum capacity performance compared to the optimal sum capacity methods, but the minimum interference method loses 0.96 bits/s/Hz. Therefore, it achieves a better trade-off between two coexisting systems.

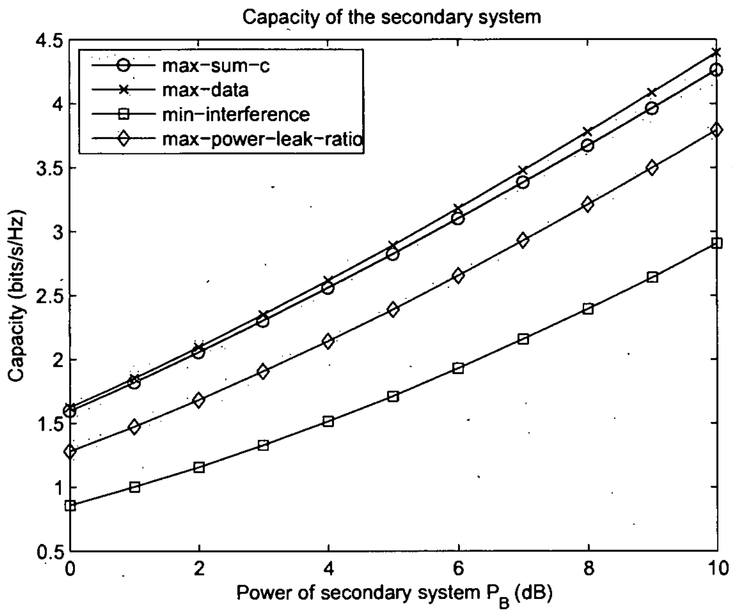


Figure 5.12: *Capacity of the secondary system for single antenna selection schemes*

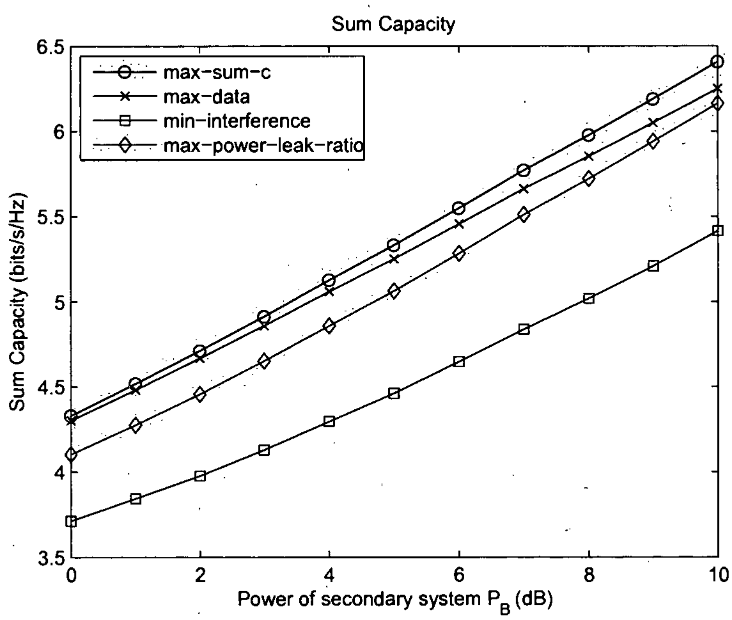


Figure 5.13: *Sum capacity for single antenna selection schemes*

5.5.2 Multiple antenna selection

In this subsection, we will study the performance of the multiple-antenna selection methods. We assume that interference factor r_{BA} equals to 1.0 and the transmitter power of the secondary

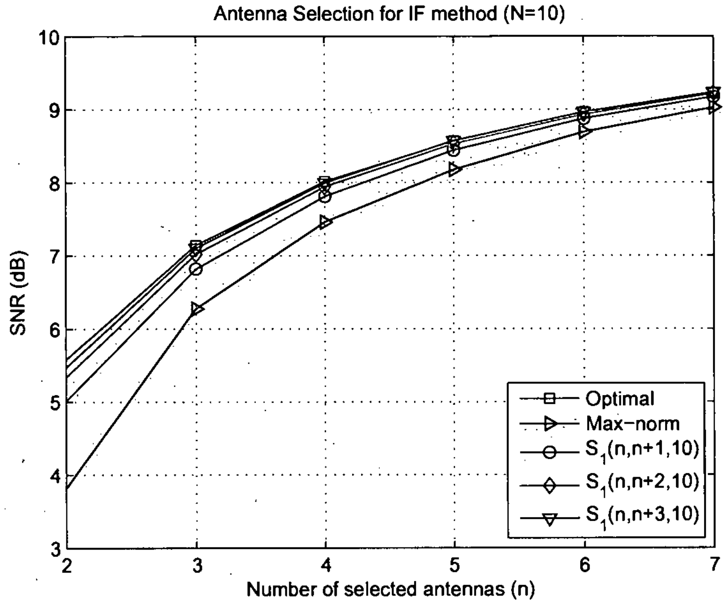


Figure 5.14: *Comparison of SNR (System B) for antenna selection with IF method*

system $P_B = 0$dB. For IC beamforming, the interference constraint $J \leq 0.2$. There are a total of $N = 10$ antennas in the transmitter of the secondary system, and we compare the SNR performance of the secondary system for the optimal selection, maximum norm, and subset optimal selection strategies where the number of RF chain $n \in \{2, 3, \cdots, 7\}$. The results for IF and IC are shown in Figure 5.14 and Figure 5.15 respectively.

It is clear that the maximum norm method achieves the worst performance for both IC and IF precoding approaches. It is because this method does not consider the cross correlation coefficient ρ or angle θ between the interference channel and the data channel. Our proposed subset method performs closer to the optimal method. For IF precoding, considering only 2 more antennas than the RF chain size (i.e. $t = n + 2$) will lead to near to optimal SNR performance; for the IC approach, adding just 1 antenna more than the RF chain size can almost achieve the optimal SNR performance.

The comparison of computational complexity, required feedback information, and performance for different multiple antenna selection strategies are listed in Table 5.2 in which 4 antennas will be selected from 10 available antennas. The size of antenna subset t is set to 6 since 2 more antennas than the RF chain size will lead to near to optimal SNR performance according to the simulation results in Figure 5.14 and Figure 5.15. We assume that the optimal strategy

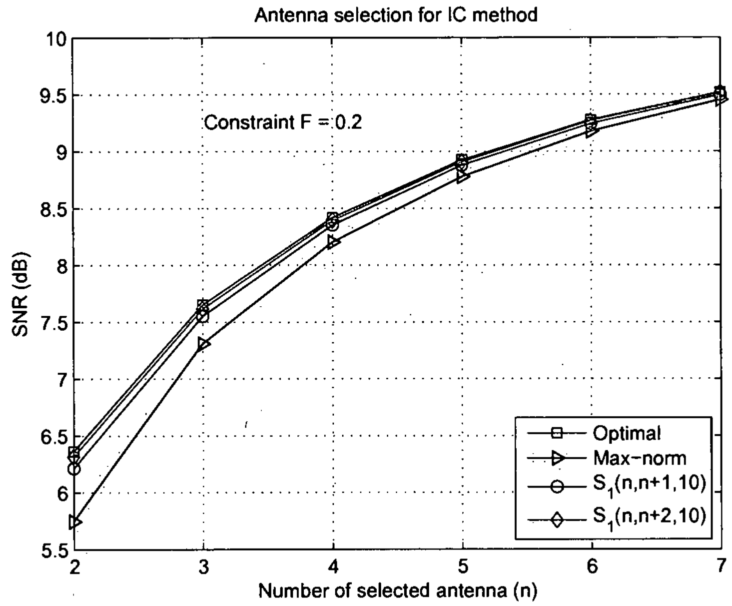


Figure 5.15: *Comparison of SNR (System B) for antenna selection with IC method*

achieves 100% SNR performance, and the other strategies are compared to the optimal value. The computational complexity is the number of metrics $\overline{G}$ to be computed, and the required CSI indicates the number of complex channel realisations. For example, to select 4 antennas from 10 available antennas it means there are $K = \binom{10}{4} = 210$ antenna combinations. If optimal selection strategy is used, 210 data channel gains $\overline{G}$ should be calculated to obtain the maximum one. Furthermore, to calculate these channel gains in the transmitter of the secondary system, all the data CSI and interference CSI for 10 antennas should be fed back by users of the primary system and secondary system respectively. Therefore, the required CSI is 20 (the number of complex values).

	Computational complexity	Required CSI	IF SNR	IC SNR ($F \leq 0.2$)
Optimal	210	20	100%	100%
Max-norm	—	8	87.5%	95.1%
$S_1(4, 6, 10)$	15	12	97.8%	99.4%
$S_2(4, 2, 6, 10)$	6	12	97.2%	99.2%

Table 5.2: *Comparison of antenna selection strategies* $n = 4, N = 10$

We can see from this table that the maximum norm strategy can achieve acceptable performance

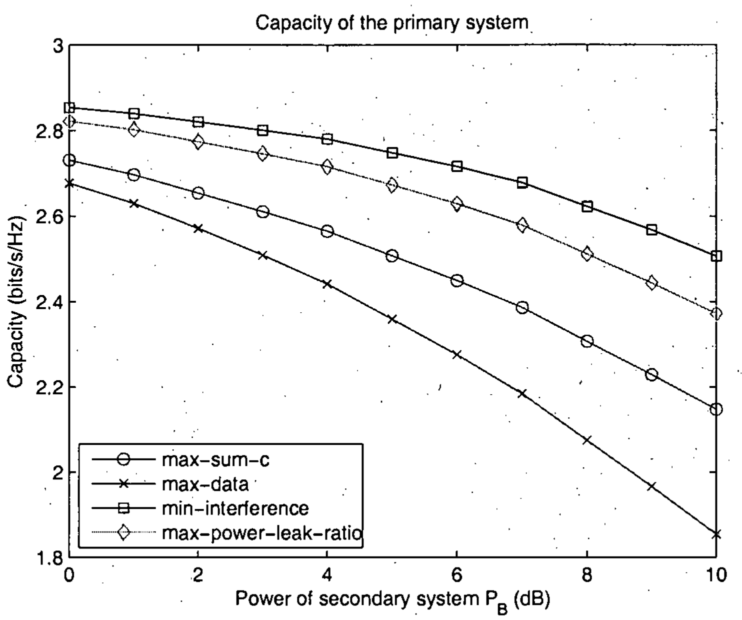


Figure 5.11: *Capacity of the primary system for single antenna selection schemes*

the case where P_B = 8dB, the maximum SLIR method outperforms by 0.82 bits/s/Hz the minimum interference method but loses 0.57 bits/s/Hz compared to the maximum data power method.

Finally, we compare the sum capacities that are presented in Figure 5.13. Although the minimum interference method is the safest method, it will lose some sum capacity performance. The maximum data method achieves near optimal sum capacity because it greatly improves the performance of the secondary system and the interference factor is not large and the transmission power of the secondary system is small. All of these lead to a small interference to the primary system although maximum data method is used. However, when the transmission power of the secondary system increases, the sum capacity performance of maximum data scheme will degrade quickly. Our proposed method improves the sum capacity compared to the minimum interference method whilst having little influence on the primary system. For P_B = 8dB, it loses 0.26 bits/s/Hz for sum capacity performance compared to the optimal sum capacity methods, but the minimum interference method loses 0.96 bits/s/Hz. Therefore, it achieves a better trade-off between two coexisting systems.

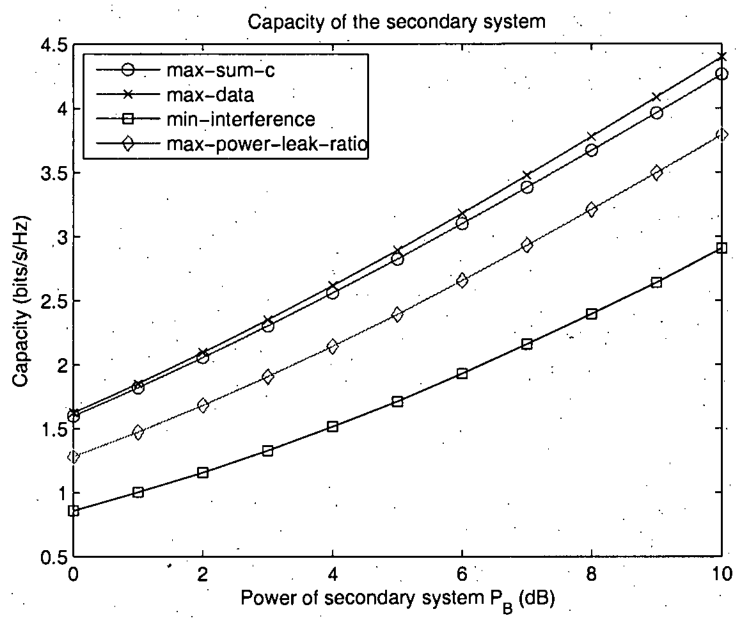
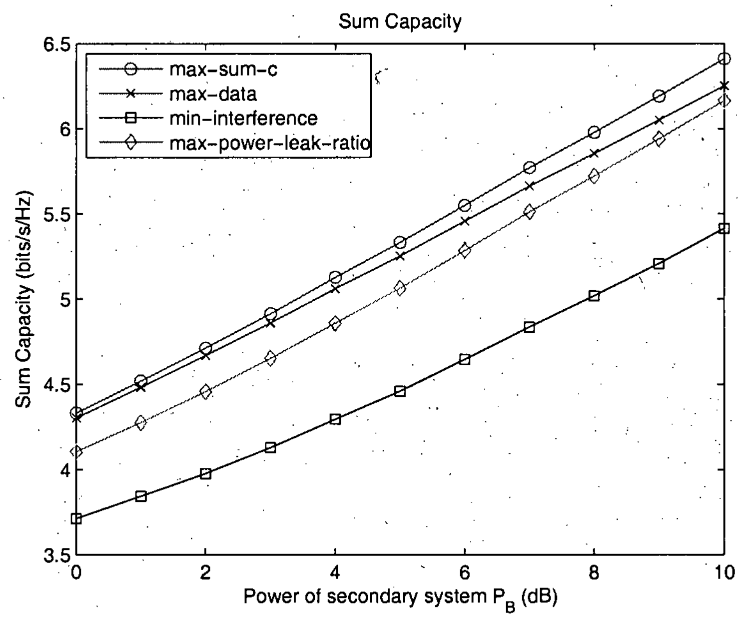


Figure 5.12: *Capacity of the secondary system for single antenna selection schemes*



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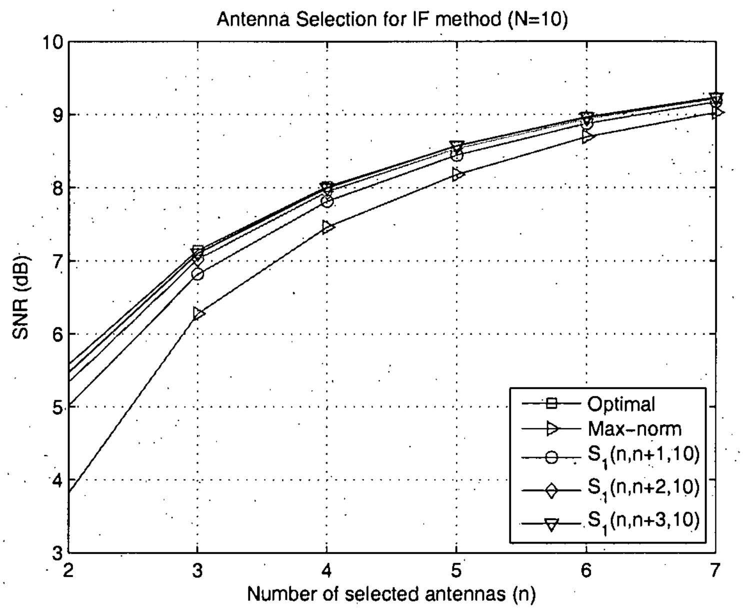

Figure 5.14: *Comparison of SNR (System B) for antenna selection with IF method*

system $P_B = 0$dB. For IC beamforming, the interference constraint $J \leq 0.2$. There are a total of $N = 10$ antennas in the transmitter of the secondary system, and we compare the SNR performance of the secondary system for the optimal selection, maximum norm, and subset optimal selection strategies where the number of RF chain $n \in \{2, 3, \cdots, 7\}$. The results for IF and IC are shown in Figure 5.14 and Figure 5.15 respectively.

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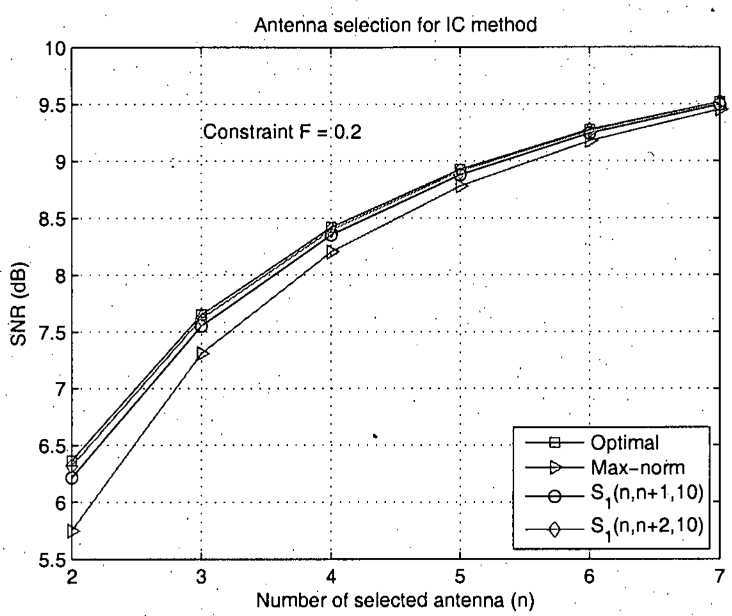

Figure 5.15: *Comparison of SNR (System B) for antenna selection with IC method*

achieves 100% SNR performance, and the other strategies are compared to the optimal value. The computational complexity is the number of metrics $\overline{G}$ to be computed, and the required CSI indicates the number of complex channel realisations. For example, to select 4 antennas from 10 available antennas it means there are $K = \binom{10}{4} = 210$ antenna combinations. If optimal selection strategy is used, 210 data channel gains $\overline{G}$ should be calculated to obtain the maximum one. Furthermore, to calculate these channel gains in the transmitter of the secondary system, all the data CSI and interference CSI for 10 antennas should be fed back by users of the primary system and secondary system respectively. Therefore, the required CSI is 20 (the number of complex values).

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$S_2(4, 2, 6, 10)$	6	12	97.2%	99.2%

Table 5.2: *Comparison of antenna selection strategies* $n = 4, N = 10$

We can see from this table that the maximum norm strategy can achieve acceptable performance when the IC approach is used for the secondary system; the proposed strategies, $S_1(4,6,10)$ and $S_2(4,2,6,10)$ with very low computational complexity and little CSI feedback requirements achieve almost optimal performance, especially when IC beamforming is used.

5.6 Conclusion

Multiple antenna technologies have been used in coexisting environments to cancel the co-channel interference. However, there are two drawbacks. One is the cost of the RF chain, the other is the feedback channel, which have limited the application of multiple antennas. In this chapter, we apply antenna selection algorithms to solve these problems. Two scenarios are discussed: one is the single-antenna selection scenario and the other is the multiple-antenna selection scenario.

The key point for the single-antenna selection scenario is that the co-channel interference can not be avoided. We first analyse the performance of the minimum interference selection strategy in which space diversity is used to reduce the CCI. The maximum data power strategy can be used to achieve the best performance for the secondary system, however it can degrade the primary system. The maximum sum capacity method achieves the best throughput trade-off between two coexisting radio system. Then we propose a novel maximum signal power to leak interference power ratio (SLIR) strategy, which considers the leaked interference and data power at the same time. The simulation results show that our proposed method achieves a better trade-off between two coexisting systems and obtain good overall capacity performance.

The other scenario we discussed is for multiple-antenna selection, in which the linear precoding methods IF and IC are used. After presenting the optimal selection strategy which is difficult to implement due to the cost of hardware and the maximum norm selection strategy which loses some performance, we propose a subset optimal multiple antennas selection strategy. The key idea of our proposed method is to only explore a subset of all the available antennas. The simulation results show that our proposed method greatly reduces the computational complexity whilst achieving the near optimal performance.

Chapter 6
Conclusion

The aim of this thesis is to apply multiple antenna technologies in cognitive radio environments in order to control and mitigate the downlink interference caused by the coexistence of multiple radio systems in an overlapping frequency band. Our interest focuses on the downlink of multiple antenna single output (MISO) cognitive radio where only base stations employ multiple antennas because of the cost of equipment and power assumptions. We firstly summarise our results and highlight the contributions of this thesis in Section 6.1; and then the limitations of this work and possible research topics are discussed in Section 6.2.

6.1 Summary and thesis contributions

Cognitive radio is proposed in the beginning of this century [3] to meet the quickly increasing requirements of throughput for wireless applications, better quality of service (QoS) for users, and reliability of communications for mobile subscribers. It is expected to solve the problems of the lack of spectrum resource for emerging applications and insufficient usage of already allocated spectrum resources in order to increase the spectrum efficiency. This technology can also give regulators, such as FCC , Ofcom, etc, the flexibility of managing the spectrum resource to allow more operators to dynamically share the spectrum resource and possibly reduce the costs of spectrum for individual operators. Generally, cognitive radio is a technology which allows multiple radio systems to coexist in overlapping spectrum and intelligently control and manage the interference between coexisting radio systems to guarantee the performance of individual radio systems according to their requirements. Multiple antenna technologies can be used in cognitive radio systems to deal with the interference and improve the performance of coexisting radio systems because it can supply multiplexing gain and diversity gain due to the independent channel fading between different pairs of antennas. In this thesis, the MISO cognitive radio scenarios are considered and discussed. The interest of this thesis focuses on the interference control and management of downlink coexisting radio systems which are synchronised and have same time frame structure.

Chapter 2 describes the basic background of this thesis, including cognitive radio, interference channels, and multiple antenna technologies. For cognitive radio systems, the conception and motivation, benefits and disadvantages along with interesting research topics of cognitive radio system are firstly presented. And then based on these discussion, this chapter analyses the interference scenarios in both synchronised and unsynchronised coexisting radio systems. Two applications of cognitive radio, femtocells and self-organising networks, are also introduced. The second part of this chapter is a brief overview of the interference channel, which is a help for understanding the interference model of cognitive radio systems. Multiple antenna technologies are discussed in the third part of this chapter. Since our focus is interference mitigation for cognitive radio in this thesis, only related topics in MIMO technology are reviewed.

Single system beamforming, in which radio systems independently precode according their own channel state information for the downlink of MISO-CR, are analysed and discussed in Chapter 3. The system model of MISO-CR is presented first. The MRT achieves the best performance for one single system but causes serious interference to the other system; the ZF causes no interference but loses some diversity gain. Based on these, the optimal interference free (IF) precoding vector, which is the optimal combination of the ZF precoding vectors is presented and analysed. It cancels the interference to the other system and obtains the remaining diversity gain. Sometimes the primary system can afford some interference. To utilise this headroom to improve the performance of the secondary system, an optimal interference-constrained (IC) precoding method is proposed. The IC precoding vector is the optimal linear combination of the IF vector and an interference vector, and it can achieve the best performance for the secondary system with the interference constraint of the primary system. The comparison of these single system approaches gives us a clear view of their relationship.

The single system beamforming methods do not consider the channel state information (CSI) of other coexisting systems, therefore leading to a decrease in performance. In Chapter 4, the joint beamforming schemes for the downlink of MISO-CR, where both transmitters know all the CSI, are analysed and discussed. According to the interference-constrained (IC) method proposed in Chapter 3, we simplify the MISO-CR channel to a "cognitive interference channel", where the interference channel and the data channel can affect each other by adjusting the interference coefficients.

These proposed joint beamforming methods in Chapter 4 aim to solve two types of issues. The first one is that one radio system has higher priority to access the spectrum, and the other is two systems have the same priority. For the first problem, an adaptive beamforming method based on single system approaches is proposed, which can dynamically adjust according to the CSI. The secondary system only uses the spectrum when the channel conditions of the primary system are good enough or very bad, therefore, the performance of the secondary system is improved while the QoS of the primary system has not been affected. Thus it can maximise the performance of the secondary system while the primary system has a minimum requirement of SINR. The secondary system can sufficiently utilise the "headroom" of the primary system to improve its performance using this method. Moreover, a low computation complexity (LCC) adaptive method is also presented to avoid iterative algorithms for finding the best solution without losing much performance. When the two systems have the same right to access the spectrum, we try to achieve the optimal overall performance via joint beamforming. The sum mean squared error (SMSE) and sum throughput metrics are selected as our performance criteria in this chapter. Due to the non-linearity and non-convexity of the cost functions, both closed form and numerical globally optimum solutions are very difficult to obtain. Our proposed sub-optimal methods utilise amended iterative methods to find better performance trade-offs for the two systems. In addition, considering that the transmitter can only transmit integer modulation schemes in real systems, a beamforming method that can achieve the sub-optimal discrete sum throughput is proposed. This method is based on the sub-optimal continuous sum throughput performing, and has better performance than simply rounding down the data rate result of the sub-optimal continuous sum throughput technique. Generally, joint beamforming schemes achieve the best performance trade-off between two radio systems and maximally increase the spectrum efficiency at the cost of full CSI.

The cost of RF chains and requirement for feedback information limit the application of beamforming. The antenna selection technique, which can reduce the hardware complexity whist keeping much of the benefit of multiple antennas, are analysed and discussed in Chapter 5. The single-antenna selection strategy is the cheapest solution since only one RF chain is needed in the transmitter side and usually the computational complexity of such strategies is linearly proportional to the number of available antennas. However the interference to the primary system normally is practically unavoidable for the single antenna selection schemes. The minimum interference method uses the antenna selection gain to the interference power and keeps acceptable performance for the primary system; for the maximum data power method, the selection gain is used to obtain better performance of the secondary system without considering the interference to the primary system. Our proposed method, maximum signal to leak interference ratio (SLIR), considers two radio systems at the same time. Our aim is to find an antenna which causes little interference to the primary system whilst obtaining high data channel gain. It has been shown that our proposed method loses a small mount of capacity for the primary system but greatly improves the capacity of the secondary system compared to the safest minimum interference method. It achieves a good performance trade-off between two coexisting radio systems. By contrast, multiple-antenna selection techniques can fully control the interference and improve the performance of the secondary system. The optimal solution achieves the best performance at the cost of significant computational complexity and full CSI. The maximum norm method just computes the signal amplitudes for the transmit antennas and has very low computational complexity. Moreover, it can be processed in the terminal and only the CSI of selected antennas is required. However, some performance will be lost due to not considering the correlation between the interference channel and data channel. To decrease the computational complexity whilst keeping near optimal performance, the subset optimal strategies are proposed. The key idea of such strategies is to decrease the size of explored antenna set according to its amplitude. Furthermore, because the process of selecting a subset can be performed at the terminal side, the requirement of the feedback channel is reduced. It has been shown that the maximum norm method achieves worst performance for both IC and IF precoding approaches, and the proposed subset methods perform close to the optimal method with very low computational complexity and low CSI requirements, especially when IC beamforming is used.

6.2 Limitations and further work

Throughout the research work in this thesis, we have several basic assumptions which may not be realistic: 1) The base station knows the exact channel state information according to the requirements of algorithms; 2) Both the primary system and secondary system are single user systems; 3) The coexisting radio systems are synchronised and only downlink interference is considered;

To relax these assumptions we can extend our research topics in following:

- **Measurement and feedback of CSI mechanism in cognitive radio.** As we discussed in Chapters 3, 4, and 5, the technologies used in this thesis need the channel state information, normally at least including its own data channel and interference channel. The CSI for data channel can be obtained from its own terminals and fed back to base station through its signalling channel. However, CSI for the interference channel is quite difficult to obtain. This issue can be divided into two separate tasks, estimation of the CSI and feedback of the information to the base station of the other radio system. Usually, the CSI is estimated by terminals when the base station transmits preambles or pilots which are already known by terminals. It means the terminals must know the synchronisation signals of the base station of the other radio system and the synchronisation signals of coexisting radios must be decoded. Further more, there is no direct link between terminals of one radio system and base stations of the other coexisting system, which is needed to feed back the interference CSI. All of these communication paths are not supported by current radio systems and an effective mechanism for obtaining and feeding back CSI in cognitive radio environments need to be developed.

- **Performance effect of inexact CSI in MISO cognitive radio.** The inexact CSI can influence the performance of multiple antenna algorithms. This may be caused by estimation error in terminals, throughput limitations for the feedback channel, and the time delay of the feedback. Some papers have already discussed the performance effect of inexact CSI in multiple input and multiple output radio system [76] [77] [78] [79] [80]. Based on this, the further research is needed to study the impact on MISO cognitive radio environments. This also gives a guideline of how many bits are needed for feeding back CSI in order to achieve a satisfactory effect of interference mitigation when designing a realistic coexisting radio system.

- **Multiple user cognitive radio.** To simplify the problem, only single user radio systems are discussed in this thesis. However, realistic radio systems usually have more than one user. There are some papers which discussed the multiple user cognitive radio environments, mainly including the multi-access channel in cognitive radio networks [81] [82], power control and rate allocation in cognitive radio [83] [84], and resource allocation in multiuser cognitive radio environments [85]. It is quite interesting to apply the beamforming methods discussed and proposed in this thesis to multi-user MISO-CR scenario, in which the coexisting radio systems may be OFDMA-based. The users are allocated resources, including spectrum, power, etc., according to the interference channel and data channel CSI to achieve the optimal performance for coexisting radio system without great impact on the other radio system. This procedure also needs to consider the computational complexity of scheduling algorithms and the fairness issues between the coexisting radio systems and the users in each individual radio system. The user selection gain and scheduling gain may be obtained and the overall performance of cognitive radio can be improved.

- **Study on complex interference scenarios in cognitive radio.** Only the downlink interference from one base station of one sharing radio system is considered in this thesis. The coexisting radio systems are synchronised and only one coexisting radio system is considered, just like the discussion in Chapter 2. But in realistic applications of cognitive radio, the situation is different. One scenario is femtocell network. Since the femtocell base stations are installed by users, many femto radio systems may coexist in a small area, and it means lots of radio systems share the spectrum. Therefore each radio system has to suffer serious interference from multiple interference sources. The coexistence studies for femtocell applications will be a big challenge and have to be carefully considered in order to use femto radio systems in real networks. Another important scenario is the coexistence between radio systems which have different radio access technologies. These radio systems usually have different frame structure, then synchronisation between these radio system usually is impossible. Therefore, in this scenario the interference may come from base stations and users. Using multiple antenna technology to manage CCI in this scenario will be very useful and complex.

Appendix A
Publications

- Jun Zhou and John Thompson, "Antenna selection for MISO cognitive radio", submitted to IEEE Transactions on Vehicular Technology for possible publication;

- Jun Zhou and John Thompson, "Linear Precoding for the Downlink of Multiple Input Single Output Coexisting Wireless Systems", IET Communications, Volume 2, Issue 6, pp. 742 - 752, July 2008.

- Jun Zhou, John Thompson, and Ioannis Krikidis, "Multiple Antenna Selection for Linear Precoding MISO Cognitive Radio", IEEE Wireless Communications and Network Conference (2009), Budapest, Hungary, 5-8 April 2009.

- Jun Zhou and John Thompson, "Single-antenna Selection for MISO Cognitive Radio", Cognitive Radio and Software Defined Radios: Technologies and Techniques, 2008 IET Seminar on, London, UK, 18-18 September, 2008.

- Jun Zhou and John Thompson, "MISO Aided Linear Interference Mitigation for Cognitive Radio", Smart Antennas and Cooperative Communications, 2007 IET Seminar on, London, UK, 22-22 October, 2007.

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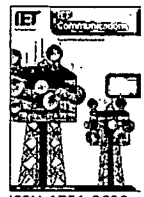

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Linear precoding for the downlink of multiple input single output coexisting wireless systems

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Abstract: Coexisting radio systems, often called cognitive radio (CR), have attracted much attention because of the lack of spectrum resources and the low usage statistics of existing spectrum allocations. Interference suppression and cancellation are seen as key technologies for enabling coexisting systems, and the application of multiple antennas might be one solution to tackle interference. Linear vector precoding for downlink of multiple input single output CR systems is addressed. The maximum ratio transmission, zero forcing, optimal interference-free, and optimal interference-constrained (IC) precoding algorithms in the sense of minimum mean squared error (MMSE) are presented. Then the authors compare and analyse these algorithms under different channel assumptions. The simulation results show that the proposed IC precoding algorithm can maximise the utilisation of multiple antennas and greatly improve the system performance.

1 Introduction

Cognitive radio (CR), proposed in 2000 by Joe Mitola [1], has attached much attention because of the lack of radio spectrum resources and the low usage statistics of existing spectrum allocations [2, 3]. The key idea of CR is the coexistence of multiple radio systems in overlapping frequency and/or time slots. Normally these two radio systems are called the primary and the secondary systems (coexisting system), respectively.

CR can be classified into two categories according to the interference models. The first one is the initial idea of the CR, called 'ordinary' CR, which refers to the coexisting system only utilising those frequency or time slots in which the signals of the primary system cannot be detected so that the secondary system will not be affected by the primary system. In this concept, the radio resource has only two values, reusable and un-reusable, depending on whether co-channel interference exists [4]. However, in real systems, apart from co-channel interference because of the reuse of spectrum, there exists other interference to radio systems, such as environmental noise, thermal noise and so on. Therefore the co-channel interference can be seen as a kind of 'noise' and it may be controlled. Whether and how the radio resource can be reused depends on the total power of the noise. According to this, the second type of CR is the extension of the initial concept of CR, called 'general' CR(or greedy CR), in which all the systems utilise the frequency or time slots at the same time, whether there is interference or not. Thus, ordinary CR is a special case of 'general' CR where the co-channel interference does not exist in the secondary user side. However, no matter which CR is used, the primary system has to suffer co-channel interference from the secondary system therefore degrading its performance. How to eliminate the interference is the key factor in coexisting radio systems.

Multiple antenna technologies may be a potential solution for mitigating interference in coexisting environments. It is one of the most important technological breakthroughs in recent years. This technology employs multiple antennas in the transmitter side and/or receiver side to achieve great improvements in system capacity and performance [5, 6]. Generally, multiple antennas can supply multiplexing gain, diversity gain and co-channel interference suppression to the wireless system because of the independent channel fading between different pairs of antennas.

Consider the downlink of a coexisting environment with multiple antennas. Since complex equipment cannot be used in the terminal side because of cost and power consumption, multiple antennas are often employed only in the base station side. Vector precoding technology is therefore used to improve system performance. Clearly, it is a typical multiple input single output (MISO) interference channel (IFC). In the case where the two systems know all the transmitted signals, the dirty paper coding (DPC) is seen as the optimal approach for the sum capacity performance [7, 8]. However, our major interests in CR system are not the sum capacity since the two systems have different priorities, and the particular challenge of the CR is that both the transmitters and receivers are distributed and may be unable to coordinate with each other. Thus we consider approaches which do not need knowledge of the other system's transmitted signals. In Larsson and Jorswieck's work [9, 10], they discussed the capacity region bound for MISO IFC based on game theory, but did not give the vectors precoding algorithms for a given performance requirement for the primary system. The maximum ratio transmission (MRT) method in [11] maximises the received signal-to-noise ratio (SNR), but it does not consider the interference to the other radio system and therefore degrades its performance. The zero-forcing (ZF) method, which comes from multiple input and multiple output multi-user detection (MIMO-MUD) techniques [12], perfectly mitigates the interference to other radio systems. However, it may degrade the power of desired signals and lose some of the diversity gain of the channel. The method proposed by Li *et al.* [13] for secret communications in MISO case maximise the secrecy capacity, which is equal to the difference between message channel capacity and interference channel capacity. In this method the interference power might be small in some cases, but it also might be strong when doing this leads to a great performance increase for the desired user. However, this is not allowed for the CR environment since usually the performance of the primary system should be guaranteed and the interference power should be controlled below a certain value.

In this paper, our focus is on finding linear precoding solutions for the downlink of coexisting environments. We consider the single-system case in which independent radio systems only know their own channel state information for the two terminals. First, we present and prove the optimal interference-free (IF) approach, which is equivalent to TxZF in MIMO system in [14]; then a novel linear precoding approach, the interference constrained (IC) linear precoding algorithm, is proposed when the primary system can suffer controlled co-channel interference; moreover, we compare these linear approaches under different assumptions of channel state information. The simulation results of these approaches under variable channel environments show that the proposed precoding algorithm can maximise the utilisation of multiple antennas and greatly improve the system performance under reasonable constraints.

The rest of this paper is organised as follows. Section 2 describes the system model and basic assumptions of this paper. The known and proposed precoding approaches are presented in Section 3. Section 4 compares these approaches and gives a summary. Simulation results comparing the algorithms under various conditions are given in Section 5, and finally, we conclude our paper in Section 6.

Notation: All boldface letters indicate vectors (lower case) and matrices (upper case). The notation $\mathrm{Tr}(\boldsymbol{A})$, $\lambda(\boldsymbol{A})$, $\mathrm{rank}(\boldsymbol{A})$, $\boldsymbol{A}^{\mathrm{T}}$, $\boldsymbol{A}^{*}$ and $\boldsymbol{A}^{\mathrm{H}}$ are the trace, the eigenvalue, rank, transpose, conjugate and conjugate transpose of $\boldsymbol{A}$. The scalar $\|\boldsymbol{b}\| = \left(\sum b_i b_i^*\right)^{1/2}$, is the norm or length of vector $\boldsymbol{b}$, and the norm of a scalar x is presented as $|x|$. $E[\cdot]$ denotes the mathematical expectation.

2 System models

A block diagram of the MISO-CR system for downlink transmission is shown in Fig. 1. Suppose that there are two radio systems, systems A and B. System A is the primary system and system B is the secondary system. It is representative of a typical coexisting environment. The base stations of system A and system B have N_A and N_B antennas, respectively. We assume that $N_A \geq 2$ and $N_B \geq 2$. In order to avoid the issue of designing space-time precoders in the transmitters, we assume that the messages of system A and system B are scalars, expressed as s_A and $s_B \in \mathbb{C}$. They are multiplied by the vectors $\boldsymbol{v}_A \in \mathbb{C}^{N_A \times 1}$ and $\boldsymbol{v}_B \in \mathbb{C}^{N_B \times 1}$, then transmitted over frequency non-selective radio channels.

The received signals y_A and y_B are

$$y_A = \boldsymbol{h}_{AA}\boldsymbol{v}_A s_A + r_{BA}\boldsymbol{h}_{BA}\boldsymbol{v}_B s_B + z_A \quad (1)$$

$$y_B = \underbrace{\boldsymbol{h}_{BB}\boldsymbol{v}_B s_B}_{\mathrm{I}} + \underbrace{r_{AB}\boldsymbol{h}_{AB}\boldsymbol{v}_A s_A}_{\mathrm{II}} + \underbrace{z_B}_{\mathrm{III}} \quad (2)$$

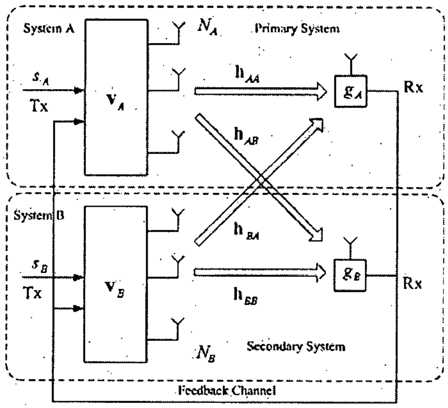

Figure 1 *Block diagram of MISO-CR system showing the base stations of system A and B transmitting to users A and B*

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where part I is the desired signal, part II the co-channel interference (CCI) and part III the additive noise. The row vectors $\boldsymbol{h}_{AA} \in \mathbb{C}^{1\times N_A}$, $\boldsymbol{h}_{AB} \in \mathbb{C}^{1\times N_A}$, $\boldsymbol{h}_{BB} \in \mathbb{C}^{1\times N_B}$ and $\boldsymbol{h}_{BA} \in \mathbb{C}^{1\times N_B}$ are channel vectors, whose elements are defined for different channel models. Moreover, define $\boldsymbol{h}_{AA}$, $\boldsymbol{h}_{BB}$ as message channels, and $\boldsymbol{h}_{AB}$, $\boldsymbol{h}_{BA}$ as interference channels with scaling channel weight r_{AB} and r_{BA}, respectively. These two scales can allow for interference reduction because of waveform design, frequency overlap etc. When either r_{AB} or r_{BA} are equal to zero, it is an ordinary CR scenario; otherwise it is the general CR case. The noise terms z_A and z_B are independent, identically distributed (i.i.d.) complex Gaussian random variables with zero mean. Their covariances are σ_A^2 and σ_B^2, respectively.

In the receivers, the received signals are multiplied by the complex scalars g_A and g_B, respectively. Then a slicer is applied to demodulate the transmitted data. The receivers also estimate their corresponding data channel and interference channel information, and according to different applications, feedback this channel information to the transmitters, as shown in Fig. 1.

We assume that the average transmit powers are fixed

$$E\left[\|\boldsymbol{v}_A s_A\|^2\right] = E\left[\|s_A\|^2\right]\mathrm{Tr}\left(\boldsymbol{v}_A\boldsymbol{v}_A^{\mathrm{H}}\right) = P_A$$
$$E\left[\|\boldsymbol{v}_B s_B\|^2\right] = E\left[\|s_B\|^2\right]\mathrm{Tr}\left(\boldsymbol{v}_B\boldsymbol{v}_B^{\mathrm{H}}\right) = P_B$$

Without loss of generality, define

$$E\left[|s_A|^2\right] = P_A \text{ and } E\left[|s_B|^2\right] = P_B \tag{3}$$

then

$$\mathrm{Tr}(\boldsymbol{v}_A\boldsymbol{v}_A^{\mathrm{H}}) = \mathrm{Tr}(\boldsymbol{v}_B\boldsymbol{v}_B^{\mathrm{H}}) = 1$$

The received signal-to-interference-noise ratio (SINR) for system B is

$$\mathrm{SINR}_B = \frac{P_B\boldsymbol{v}_B^{\mathrm{H}}\boldsymbol{h}_{BB}^{\mathrm{H}}\boldsymbol{h}_{BB}\boldsymbol{v}_B}{r_{AB}^2P_A\boldsymbol{v}_A^{\mathrm{H}}\boldsymbol{h}_{AB}^{\mathrm{H}}\boldsymbol{h}_{AB}\boldsymbol{v}_A + \sigma_B^2} \tag{4}$$

The normalised mean squared error for system B is defined as

$$\mathrm{MSE}_B^{\pi} = E\left[|s_B - g_B y_B\|^2\right]P_B \tag{5}$$

For fixed precoding vectors $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$, the minimum normalised mean squared error (MMSE) for system B is

$$\mathrm{MMSE}_B^{\pi} = \frac{r_{AB}^2P_A\boldsymbol{v}_A^{\mathrm{H}}\boldsymbol{h}_{AB}^{\mathrm{H}}\boldsymbol{h}_{AB}\boldsymbol{v}_A + \sigma_B^2}{P_B\boldsymbol{v}_B^{\mathrm{H}}\boldsymbol{h}_{BB}^{\mathrm{H}}\boldsymbol{h}_{BB}\boldsymbol{v}_B + r_{AB}^2P_A\boldsymbol{v}_A^{\mathrm{H}}\boldsymbol{h}_{AB}^{\mathrm{H}}\boldsymbol{h}_{AB}\boldsymbol{v}_A + \sigma_B^2} \tag{6}$$

where

$$\hat{g}_B = \frac{\sqrt{P_B}\boldsymbol{v}_B^{\mathrm{H}}\boldsymbol{h}_{BB}^{\mathrm{H}}}{P_B\boldsymbol{v}_B^{\mathrm{H}}\boldsymbol{h}_{BB}^{\mathrm{H}}\boldsymbol{h}_{BB}\boldsymbol{v}_B + r_{AB}^2P_A\boldsymbol{v}_A^{\mathrm{H}}\boldsymbol{h}_{AB}^{\mathrm{H}}\boldsymbol{h}_{AB}\boldsymbol{v}_A + \sigma_B^2}$$

Since systems A and B are symmetrical, the equations for SINR_A, MSE_A^{π}, MMSE_A^{π}, and $\hat{g}_A$ are similar to those for system B.

3 Single-system algorithms

The single-system algorithms are used when the two systems cannot exchange their channel state information. Each system independently precodes depending on their own channel state information for users A and B. Sometimes, one radio system even is not aware of the existence of other radio system. In this section, we consider precoding methods for system B (the secondary system) and fix the precoding vector for system A (the primary system). Firstly we introduce the known precoding approaches, MRT and ZF. Then the optimal IF and optimal IC techniques are presented and their properties proved. Each approach is discussed not only considering the performance of system B, but also the considering the effect to the other coexisting system (system A). In these approaches, we usually assume that the base station of system B knows the channel state information of the interference channel and/or its own data channel through feedback from the terminals. Although these linear precoding methods are based on single user CR system, we can expend them to multiple user system using other multiplexing techniques, such as CDMA, FDMA or TDMA.

3.1 Maximum ratio transmission

Typically, there are two kinds of known precoding approaches for MISO systems. One is MRT or TxMF [11, 14]. This method maximises the received desired signal power subject to a transmitter power constraint. For system B, the problem is described as follows

$$\boldsymbol{v}_B^{\mathrm{MRT}} = \arg\max_{\{\boldsymbol{v}_B\}}|\boldsymbol{h}_{BA}\boldsymbol{v}_B|^2 \text{ s.t.: } \mathrm{Tr}(\boldsymbol{v}_B\boldsymbol{v}_B^{\mathrm{H}}) = 1$$

The solution of this approach is proportional to the conjugate transpose of the normalised message channel

$$\boldsymbol{v}_B^{\mathrm{MRT}} = \frac{\boldsymbol{h}_{BB}^{H}}{\|\boldsymbol{h}_{BB}\|} \tag{7}$$

This method provides the best performance for system B, but without considering the interference to system A. According to (4) and (6), it achieves the maximum SNR and minimum MSE for user B [11, 14] for a given precoding vector of primary system. The maximal diversity gain against the fading channel is also obtained. In addition, only the message channel information for system B is needed. The main drawback of MRT in coexisting environments is that it may greatly degrade the performance of system

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A because of the interference coming from system B. According to (1), (3)and (7) the interference power to system A, I_A^{MRT}, can be written as

$$\begin{aligned} I_A^{\mathrm{MRT}} &= E\left[\left|r_{BA}\boldsymbol{b}_{BA}\boldsymbol{v}_B^{\mathrm{MRT}}s_B\right|^2\right] = r_{BA}^2P_B\left|\boldsymbol{b}_{BA}\boldsymbol{v}_B^{\mathrm{MRT}}\right|^2 \\ &= \frac{r_{BA}^2P_B|\boldsymbol{b}_{BA}\boldsymbol{b}_{BB}^{\mathrm{H}}|^2}{|\boldsymbol{b}_{BB}|^2} \\ &= r_{BA}^2R_B^2\|\boldsymbol{b}_{BA}\|^2P_B \end{aligned}$$

where R_B is the cross-correlation coefficient, defined as

$$R_B = \frac{|\boldsymbol{b}_{BA}\boldsymbol{b}_{BB}^{\mathrm{H}}|}{\|\boldsymbol{b}_{BA}\|\|\boldsymbol{b}_{BB}\|} \tag{8}$$

3.2 Zero-forcing

The next method, named ZF [12], can perfectly cancel the interference to system A. The key idea of this method is to find a vector which is orthogonal to the interference channel, $\boldsymbol{b}_{BA}\boldsymbol{v}_B = 0$. Then, this problem can be expressed as

$$\boldsymbol{v}_B^{\mathrm{ZF}} = \text{Any } \boldsymbol{v}_B,\ \boldsymbol{v}_B \in \{\boldsymbol{v}||\boldsymbol{b}_{BA}\boldsymbol{v}|^2 = 0 \text{ and } \mathrm{Tr}(\boldsymbol{v}\boldsymbol{v}^{\mathrm{H}}) = 1\} \tag{9}$$

Define the system B interference self-correlation matrix $\boldsymbol{F}_B = \boldsymbol{b}_{BA}^{\mathrm{H}}\boldsymbol{b}_{BA}$, and then $\mathrm{rank}(\boldsymbol{F}_B) = 1$. Thus, it has one non-zero eigenvalue and $N_B - 1$ zero eigenvalues, $\lambda(\boldsymbol{F}_B) = \{\lambda_{\text{non-zero}}, 0, \ldots, 0\}$, where $\lambda_{\text{non-zero}} = \|\boldsymbol{b}_{BA}\|^2$. The eigenvectors corresponding to the zero eigenvalues of the interference self-correlation matrix form the possible solutions of the ZF method. The data channel information for system B is not needed for this approach. A direct solution of ZF is

$$\boldsymbol{v}_B^{\mathrm{ZF}} = \frac{\bar{\boldsymbol{v}}_B}{\|\bar{\boldsymbol{v}}_B\|} \tag{10}$$

and

$$\bar{\boldsymbol{v}}_B = \boldsymbol{v} - \frac{\boldsymbol{b}_{BA}\boldsymbol{v}}{\|\boldsymbol{b}_{BA}\|^2}\boldsymbol{b}_{BA}^{\mathrm{H}}\ \ \boldsymbol{v} \in \mathbb{C}^{N_B\times 1},\ \|\bar{\boldsymbol{v}}_B\| \neq 0 \tag{11}$$

Proof: From (10) it is easy to see that if $\|\bar{\boldsymbol{v}}_B \neq 0\|$

$$\mathrm{Tr}(\boldsymbol{v}_B^{\mathrm{ZF}}(\boldsymbol{v}_B^{\mathrm{ZF}})^{\mathrm{H}}) = \frac{\bar{\boldsymbol{v}}_B^{\mathrm{H}}\bar{\boldsymbol{v}}_B}{\|\bar{\boldsymbol{v}}_B\|^2} = 1$$

and for arbitrary $\mathbf{v}$ which satisfies $\|\bar{\boldsymbol{v}}_B\| \neq 0$, we have

$$\begin{aligned} \boldsymbol{b}_{BA}\boldsymbol{v}_B^{\mathrm{ZF}} &= (\boldsymbol{b}_{BA}\boldsymbol{v} - \boldsymbol{b}_{BA}\frac{\boldsymbol{b}_{BA}\boldsymbol{v}}{\|\boldsymbol{b}_{BA}\|^2}\boldsymbol{b}_{BA}^{\mathrm{H}})/\|\bar{\boldsymbol{v}}_B\| \\ &= (\boldsymbol{b}_{BA}\boldsymbol{v} - \frac{\boldsymbol{b}_{BA}\boldsymbol{b}_{BA}^{\mathrm{H}}}{\|\boldsymbol{b}_{BA}\|^2}\boldsymbol{b}_{BA}\boldsymbol{v})/\|\bar{\boldsymbol{v}}_B\| = 0 \end{aligned}$$

Then $\boldsymbol{v}_B^{\mathrm{ZF}}$ is one possible ZF solution. □

If there are more than two antennas in base station B, the solution to the ZF algorithm is not unique because there are more than one independent eigenvector corresponding to a zero eigenvalue. An arbitrary choice of the ZF vector will lead to the loss of some diversity gain due to not exploiting the data channel information of user B. How to get the optimum vector for the MISO-CR case will be discussed in next section.

3.3 Optimal IF

The optimal precoding vector in the sense of causing no interference to the primary system will provide better performance than the ZF method when there are more than two transmit antennas in the base station. We select minimising MSE as our performance criterion. The problem is described as follows

$$\begin{aligned} \boldsymbol{v}_B^{\mathrm{IF}} &\doteq \arg\min_{\{\boldsymbol{v}_B\}} \mathrm{MSE}_B^{\pi r}(\boldsymbol{v}_B) \\ \text{s.t.: } &|\boldsymbol{b}_{BA}\boldsymbol{v}_B|^2 = 0 \quad \text{and} \quad \mathrm{Tr}(\boldsymbol{v}_B\boldsymbol{v}_B^{\mathrm{H}}) = 1 \end{aligned}$$

Fix the precoding vector $\boldsymbol{v}_A$, We can see from (6) that the interference power from the primary system is fixed. Then if we maximise the scalar $\|\boldsymbol{b}_{BB}\boldsymbol{v}_B^2\|$ with the above constraints, the minimising MSE can be obtained.

Theorem 1: Let $\boldsymbol{b}_{BB}$ and $\boldsymbol{b}_{BA} \in \mathbb{C}^{1\times N_B}$, $N_B \geq 2$, be the message channel and the interference channel row vectors, respectively. If they are not linearly dependent, that is $|\boldsymbol{b}_{BA}\boldsymbol{b}_{BB}^{\mathrm{H}}| \neq \|\boldsymbol{b}_{BA}\|\|\boldsymbol{b}_{BB}\|$, the optimal precoding vector $\boldsymbol{v}_B$ maximising the $|\boldsymbol{b}_{BB}\boldsymbol{v}_B|^2$ with constraints $|\boldsymbol{b}_{BA}\boldsymbol{v}_B|^2 = 0$ and $\boldsymbol{v}_B^H\boldsymbol{v}_B = 1$ is

$$\boldsymbol{v}_B^{\mathrm{IF}} = \left[\boldsymbol{I}_{N_B} - \frac{\boldsymbol{b}_{BA}^{\mathrm{H}}\boldsymbol{b}_{BA}}{\|\boldsymbol{b}_{BA}\|^2}\right]\frac{\boldsymbol{b}_{BB}^{\mathrm{H}}}{\|\boldsymbol{b}_{BB}\|\sqrt{(1-R_B^2)}} \tag{12}$$

and

$$|\boldsymbol{b}_{BB}\boldsymbol{v}_B^{\mathrm{IF}}|_{max}^2 = |\boldsymbol{b}_{BB}|^2(1-R_B^2) \tag{13}$$

Proof: See the Appendix. □

In fact, the IF method is equivalent to TxZF method which has been discussed in point-to-point MIMO systems in [14]. Here, we adapt it to the MISO-CR scenario.

The optimal precoding vector in (12) can be seen to be based on the MRT solution, which achieves the maximal value of received desired signal power, but subtracting out the vector component which causes the interference, while maintaining the unit power constraint. This also shows that in our case the optimal combination of coefficients for the null space of $\boldsymbol{b}_{BA}$ is linearly proportional to the corresponding coefficients of the MRT solution which is the optimal combination of the signal space.

Moreover, the desired signal power, as shown in (13), is not influenced by the power of the interference channel but only depends on the cross-correlation coefficient between the channel vectors, which depends on the direction of the interference channel. The desired signal power is inversely proportional to the square of the cross-correlation coefficient, which is related to the angle between the message channel vector and the interference channel vector. When the two channels are orthogonal, this approach achieves as good performance as MRT. As the correlation coefficient tends to 1.0, the desired power decreases to 0. From (6) and (13), the normalised MMSE for system B is

$$\mathrm{MMSE}_B^{nr} = \frac{r_{AB}^2 P_A \boldsymbol{v}_A^{\mathrm{H}} \boldsymbol{h}_{AB}^{\mathrm{H}} \boldsymbol{h}_{AB} \boldsymbol{v}_A + \sigma_B^2}{P_B(1-R_B^2)\|\boldsymbol{h}_{BB}\|^2 + r_{AB}^2 P_A \boldsymbol{v}_A^{\mathrm{H}} \boldsymbol{h}_{AB}^{\mathrm{H}} \boldsymbol{h}_{AB} \boldsymbol{v}_A + \sigma_B^2} \tag{14}$$

3.4 Optimal IC

The optimal IF precoding approach has been discussed in the previous subsection. However, sometimes the primary system may be able to tolerate controlled levels of interference in practical implementation. For example, the received SNR of the primary system is much better than the required SNR for acceptable performance due to good channel conditions. Or we can increase the transmission power of primary system to obtain better SNR. In this scenario, this headroom may be used to help the secondary system achieve better performance. Keeping this in mind, an IC precoding algorithm is proposed. The problem is described as follows

$$\boldsymbol{v}_B^{\mathrm{IC}} = \arg\min_{\{\boldsymbol{v}_B\}} \mathrm{MSE}_B^{nr}(\boldsymbol{v}_B)$$
$$\text{s.t.: } r_{BA}^2 P_B |\boldsymbol{h}_{BA}\boldsymbol{v}_B|^2 \le F \quad \text{and} \quad \mathrm{Tr}(\boldsymbol{v}_B\boldsymbol{v}_B^{\mathrm{H}}) = 1$$

The only difference to the IF scheme in the previous subsection is the interference constraint, therefore this problem is also equal to maximising the scalar $|\boldsymbol{h}_{BB}\boldsymbol{v}_B|^2$ with the above constraints according to equation (6).

Theorem 2: Let $\boldsymbol{h}_{BB}$ and $\boldsymbol{h}_{BA} \in \mathbb{C}^{1\times N_B}$, $N_B \ge 2$, be the message channel and the interference channel row vectors, respectively. If they are not linearly dependent, that is $|\boldsymbol{h}_{BA}\boldsymbol{h}_{BB}^{\mathrm{H}}| \ne \|\boldsymbol{h}_{BA}\|\|\boldsymbol{h}_{BB}\|$, the optimal precoding vector $\boldsymbol{v}_B$ maximising the $|\boldsymbol{h}_{BB}\boldsymbol{v}_B|^2$ with constraints $r_{BA}^2 P_B|\boldsymbol{h}_{BA}\boldsymbol{v}_B|^2 \le F$ and $\boldsymbol{v}_B^{\mathrm{H}}\boldsymbol{v}_B = 1$ is

$$\boldsymbol{v}_B^{\mathrm{IC}} = \sqrt{1-\beta^2}\,\boldsymbol{v}_B^{\mathrm{IF}} + \beta\frac{\boldsymbol{h}_{BB}\boldsymbol{h}_{BA}^{\mathrm{H}}}{R_B}\boldsymbol{h}_{BA}^{\mathrm{H}} \tag{15}$$

where

$$\beta \in R \quad and \quad 0 \le \beta \le R_B$$

If

$$\sqrt{\frac{F}{r_{BA}^2 P_B \|\boldsymbol{h}_{BA}\|^2}} \ge R_B \tag{16}$$

then

$$|\boldsymbol{h}_{BB}\boldsymbol{v}_B^{IC}|_{\max}^2 = \|\boldsymbol{h}_{BB}\|^2 \quad \text{and} \quad \beta = R_B \tag{17}$$

else

$$|\boldsymbol{h}_{BB}\boldsymbol{v}_B^{\mathrm{IC}}|_{\max}^2 = \|\boldsymbol{h}_{BB}\|^2\sqrt{(1-\beta^2)(1-R_B^2)} + \beta R_B|^2 \tag{18}$$

and

$$\beta = \sqrt{\frac{F}{r_{BA}^2 P_B\|\boldsymbol{h}_{BA}\|^2}} \tag{19}$$

Proof: See the Appendix. □

From (17) and (18) we can see that allowing some interference to the primary system will increase the desired signal power of the secondary system. The key idea of the IC precoding approach is the trade-off between the two systems. It tries to include some contributions from the component which causes the interference to the primary system. The performance of IC depends on the cross-correlation coefficient of the interference channel vector and the message channel vector. When the interference constraint reaches a certain value, which equals the interference level caused by the MRT method, the maximal desired received power for user B is obtained. The IC method then becomes the MRT method. When the interference limits continue to increase, both the received data signal power and the interference to the primary system will not increase. On the other hand, when the permitted interference tends to zero, the IC tends to the IF method and β tends to zero.

4 Comparison of single system approaches

We introduced the single system precoding approaches, MRT, ZF, IF and IC in the four previous sections. The comparison of these approaches is presented in Table 1. The geometric relationships of the ZF, MRT, IF and IC methods are illustrated in Fig. 2. Note that for ease of understanding, the vector solutions have been drawn with different lengths in Fig. 2. In the implementation of these algorithms, however, these vectors would be normalised. The MRT approach achieves the best performance according to the message channel information. Its precoding vector can be seen as the optimal linear combination in the complex vector space $\mathbb{C}^{N_B\times 1}$. The basis vector set is the set of eigenvectors of the interference self-correlation matrix. Arbitrary vectors corresponding to the zero eigenvalues represent solutions of ZF. If the

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Table 1 Comparison of single system approaches

	Interference power to primary system	Message channel power of secondary system (G)	Required channel state information
MRT	$r_{BA}^2 R_B^2 \|\boldsymbol{h}_{BA}\|^2 P_B$	$\|\boldsymbol{h}_{BB}\|^2 P_B$	message channel
ZF	zero	$\leq G_{\mathrm{IF}}$	interference channel
IF	zero	$(1 - R_B^2)\|\boldsymbol{h}_{BB}\|^2 P_B$	message and interference channel
IC	$\leq F$	$G_{\mathrm{IF}} \leq G_{\mathrm{IC}} \leq G_{\mathrm{MRT}}$	message and interference channel

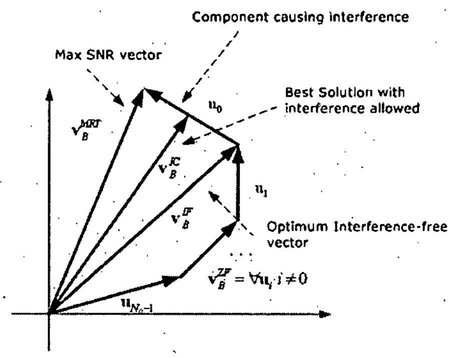

Figure 2 *Geometric interpretation of the relationship between single system approaches*

component $\boldsymbol{u}_0$ is removed, the remaining vector of the MRT precoding vector is the optimal IF solution. Moreover, the IC solution is similar to the IF solution but adds part of the vector $\boldsymbol{u}_0$, depending on the allowable interference limits. The parameter, β, called the interference coefficient, will determine the interference and desired signal power. When $\beta = 0$, there is no interference to primary system, and the algorithm becomes equal to IF; when $\beta = R_B$, the algorithm becomes equal to MRT. Another thing to notice is that all the precoding methods are affected by the cross-correlation coefficient of the interference channel and the data channel. When $R_B = 0$, the MRT, IF and IC become the same precoding method because no interference will be produced to the primary system. When R_B increases, the performance of the IF and IC, in terms of the desired signal power, will degrade and the interference caused by MRT to the primary system increases. In the worst situation, when the value of $R_B = 1$, for MRT and IC the interference power to primary system and the data power to desired system are equal. For the IF and ZF cases, both the interference power and the signal power are equal to zero.

5 Simulation and results

5.1 *Channel models and simulation conditions*

We now introduce two kinds of channel models that are used in our simulations, a Rayleigh fading channel and a more practical channel model, single cell channel.

For the Rayleigh fading channel, the elements of the channel vectors are i.i.d. complex Gaussian random variables with zero mean and unit covariance. It is used to measure the bit error ratio (BER) and MSE.

The single cell channel model [4], includes Rayleigh fading, path-loss and shadow fading. The channel vectors can be written as

$$h = \sqrt{\mathrm{SNR}_0 \left(\frac{d}{R}\right)^{-\alpha} 10^{s/10}}\, g \tag{20}$$

where SNR_0 is the median SNR when the distance between terminal and base station equals R (the cell radius); α is the path loss exponent; s models shadowing and is a real Gaussian random variable with zero mean and variance of σ_s^2; g is a complex Gaussian distributed random vector variable representing Rayleigh fading with zero mean and unit variance. The single cell channel is used to measure the Shannon capacity, as shown in (21)

$$C = \log_2(1 + \mathrm{SINR}) \tag{21}$$

We extend the single cell channel to two coexisting cell systems, as shown in Fig. 3. The two cell system use the same spectrum with overlapping area. Both the cell radii are 2.5 km, and the distance between two base stations is 1 km. For the two systems, the path loss exponent and standard deviation of shadowing is 4.0 and 3.4 dB, respectively. The terminals of both systems are randomly distributed in the cell areas. For both channel models, the noise in the receivers is set to unity (0 dB).

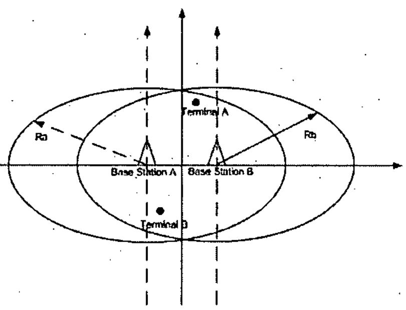

Figure 3 *Two coexisting cell system layout*

We also assume all the channels are the slow-fading wireless channels with packet-based transmission and are quasi-static over one packet length. The channel weight $r_{AB} = r_{BA} = 0.5$. For one simulation, a total of 2×10^6 packets are sent, and there are ten symbols in each packet. When measuring BER performance, both radio systems send uncoded QPSK signals.

5.2 Simulation results

For comparison of performance for these linear precoding methods, we assume that system A is the primary system, and employs two antennas in its base station. The system A is aware of the existence of system B and the ZF scheme is used by system A to avoid causing interference to system B. The transmission power (for Rayleigh fading channel) and median SNR of system A (for single cell channel) is 10.0 dB. The base station of System B has four antennas and variable transmission power. We compare the both systems' performance on this situation when different precoding approaches are used in the secondary system.

The BER results of system A and system B under the Rayleigh fading channel are shown in Figs. 4 and 5, respectively. From these two figures, we can see that MRT achieves the best performance for system B but

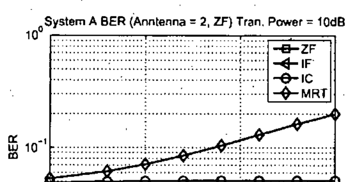

Figure 4 *BER of system A for ZF, IF, IC and MRT under Rayleigh channel (QPSK modulotion)*

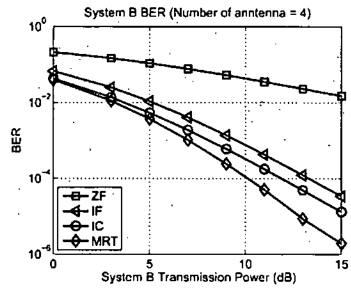

Figure 5 *BER of system B for ZF, IF, IC and MRT for the Rayleigh channel model (QPSK modulotion)*

simultaneously causes the poorest performance for system A. This is because MRT maximises the received signals for system B (full diversity gain is obtained) and produces the maximum co-channel interference to system A which greatly degrades system A's SINR.

The ZF method achieves the poorest performance for system B because the arbitrary selection of an orthogonal vector loses all the diversity gain. The performance of the IF method is superior to ZF performance because the IF method selects the optimal orthogonal vector, and obtains the maximum achievable diversity gain. It is easy to understand the IF and ZF give the same performance for system A because they do not produce any interference to it.

The interference constraint for IC is 0.2, which is 20% of the noise power. The performance of IC for system B is in-between that of IF and MRT. This is because the IC utilises part of interference vector to achieve higher SNR for system B. That also is why there is a gap between IC and IF for the performance of system A in Fig. 5. Since the constraint is defined as the allowable interference power, which is directly proportional to the transmission power, the interference coefficient for IC β is inversely proportional to the transmission power. Higher transmission power corresponds to a lower coefficient β and vice versa. Moreover, a higher interference coefficient β leads to IC performance tending to that of MRT, and a lower interference coefficient leads to an IC solution tending towards that of the IF. It explains why the IC performance tends to that of MRT when transmission power is low, and tends to that of ZF when transmission power is high.

The capacities of system A and system B with different algorithms are shown in Figs. 6 and 7, respectively. It shows the same trends as with the BER measurements for a Rayleigh fading channel shown in Fig. 4 and 5. That is, improving the performance of system B comes at the cost of degrading the capacity of system A.

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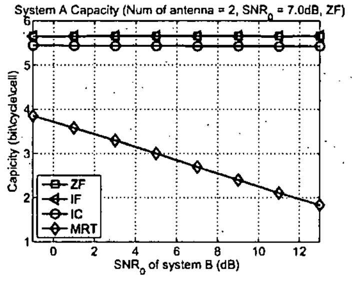

Figure 6 *Capacity of system A for ZF, IF, IC and MRT under single cell channel*

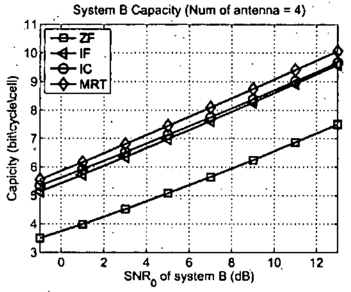

Figure 7 *Capacity of system B for ZF, IF, IC and MRT under single cell channel*

It is interesting that what will happen when we increase the number of antennas. The BER comparison of the IF and the MRT for equipment with different numbers of antennas is shown in Fig. 8. In this simulation, we compared the BER

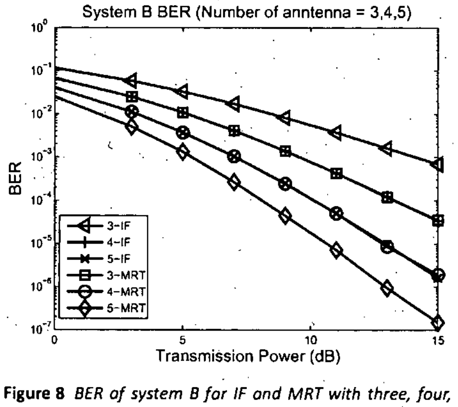

Figure 8 *BER of system B for IF and MRT with three, four, five antennas (Rayleigh fading channel, QPSK modulation)*

performance of system B for MRT and IF when it employs three, four, and five antennas in base station under Rayleigh fading channel. It is clear that the IF effectively loses the benefit of one antenna compared with the MRT. It is because the lost degree of freedom is used to remove interference to the primary system, following the theoretical analysis in [15]. It also verifies that the IF achieves the maximum achievable diversity subject to the interference constraint.

6 Conclusion

CR is an important solution to the lack of spectrum resource. Interference mitigation is likely to be a key technology for coexisting systems. In this paper, a general non-IF coexisting system, which employs multiple antennas at the base station side, is considered.

Linear vector precoding approaches can be grouped into two classes, single system approaches and joint approaches, depending on whether the two systems exchange their channel state information. In this paper we discussed the precoding method for single system scenario where one system does not know the channel state information at the other one. The MRT and ZF techniques are two well-known single system approaches. On the basis of these, we present and prove the optimal IF precoding vector, which is the optimal combination of ZF precoding vectors. It cancels the interference to other system and obtains the remaining diversity gain. When the primary system can afford some interference because of the good channel conditions, we propose an IC precoding method to utilise this headroom to improve the performance of the secondary system. The IC precoding vector is the optimal combination of IF vector and an interference vector, and it can achieve the best performance for the secondary system with a primary system interference constraint. It allows a trade-off between the two systems according to QoS requirements of the primary system. The comparison of these single system approaches via analysis and simulation gives us a clear view of their performance and relationship.

The future work may include:

(i) joint system approaches in which all the channel state information is known by both transmitters.

(ii) Analysis of the effect of inexact channel information on these linear precoding algorithms. These errors can be divided into three aspects: estimation error, capacity limits of the feedback channel and the time delay of channel estimation and feedback.

(iii) Investigation of multi-user CR channel selection strategies.

(iv) Power allocation for the multi-user CR.

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9 Appendix

9.1 Proof of Theorem 1

Proof: Let $F_B = b_{BA}^{\mathrm{H}} b_{BA}$ be the interference channel self-correlation matrix, and $G_B = b_{BB}^{\mathrm{H}} b_{BB}$ be the message channel self-correlation matrix. Since they are Hermitian matrices, they can be decomposed as

$$F_B = U^{\mathrm{H}} \mathrm{diag}\{\lambda_{BA}, 0, \ldots, 0\} U$$

and

$$G_B = M^{\mathrm{H}} \mathrm{diag}\{\lambda_{BB}, 0, \ldots, 0\} M$$

where U and M are unitary matrices [16]. Define $U = [u_0, u_1, \ldots, u_{N_B-1}]$, $P = [u_1, u_2, \ldots, u_{N_B-1}]$, and $M = [m_0, m_1, \ldots, m_{N_B-1}]$, where u_0 and m_0 are the eigenvectors corresponding the non-zero eigenvalues of F_B and G_B, respectively. Without losing generality, define $u_0 = b_{BA}^{\mathrm{H}}/\|b_{BA}\|$ and $m_0 = b_{BB}^{H}/\|b_{BB}\|$. The vector m_0 is the precoding vector of MRT. Then $\lambda_{BA} = \|b_{BA}\|^2$ and

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$\lambda_{BB} = \|\boldsymbol{b}_{BB}\|^2$. It is clear that

$$UU^{\mathrm{H}} = [\boldsymbol{u}_0, \boldsymbol{P}][\boldsymbol{u}_0, \boldsymbol{P}]^{\mathrm{H}} = \boldsymbol{u}_0\boldsymbol{u}_0^{\mathrm{H}} + \boldsymbol{P}\boldsymbol{P}^{\mathrm{H}} = \boldsymbol{I}$$

The $N_B - 1$ vectors $\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_{N_B-1}$ are independent, unit norm, and orthogonal to $\boldsymbol{b}_{BA}$ due to them corresponding to the zero eigenvalues of $\boldsymbol{F}_B$, therefore they are the basis vectors of null space of $\boldsymbol{b}_{BA}$. So if and only if the vector $\boldsymbol{v}_B$ can be represented as a complex linear combination of these vectors, $\|\boldsymbol{b}_{BA}\boldsymbol{v}_B\|^2 = 0$. Define $\boldsymbol{v}_B = \boldsymbol{P}\boldsymbol{c}$ and $\boldsymbol{c} = [c_1, c_2, \ldots, c_{N_B-1}]^{\mathrm{T}}$, then

$$\mathrm{Tr}(\boldsymbol{v}_B\boldsymbol{v}_B^{\mathrm{H}}) = (\boldsymbol{P}\boldsymbol{c})^{\mathrm{H}}\boldsymbol{P}\boldsymbol{c} = \boldsymbol{c}^{\mathrm{H}}(\boldsymbol{P}^{\mathrm{H}}\boldsymbol{P})\boldsymbol{c} = \|\boldsymbol{c}\|^2 = 1$$

Therefore we have

$$\begin{aligned}\|\boldsymbol{b}_{BB}\boldsymbol{v}_B\|^2 &= \boldsymbol{v}_B^{\mathrm{H}}(\boldsymbol{b}_{BB}^{\mathrm{H}}\boldsymbol{b}_{BB})\boldsymbol{v}_B = \boldsymbol{v}_B^{\mathrm{H}}\boldsymbol{G}_B\boldsymbol{v}_B \\ &= \boldsymbol{v}_B^{\mathrm{H}}\boldsymbol{M}^{\mathrm{H}}\mathrm{diag}\{\lambda_{BB}, 0, \ldots, 0\}\boldsymbol{M}\boldsymbol{v}_B \\ &= \lambda_{BB}\left|\boldsymbol{v}_B^{\mathrm{H}}\boldsymbol{m}_0\right|^2 = \lambda_{BB}\left|(\boldsymbol{P}\boldsymbol{c})^{\mathrm{H}}\boldsymbol{m}_0\right|^2 \\ &= \lambda_{BB}\left|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{P}\boldsymbol{c}\right|^2 \\ &\le \lambda_{BB}\left\|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{P}\right\|^2\|\boldsymbol{c}\|^2 = \lambda_{BB}\left\|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{P}\right\|^2\end{aligned}$$

When

$$\boldsymbol{c} = \frac{(\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{P})^{\mathrm{H}}}{\|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{P}\|} = \frac{\boldsymbol{P}^{\mathrm{H}}\boldsymbol{m}_0}{\|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{P}\|}$$

the equality is valid. Then

$$\boldsymbol{v}_B^{\mathrm{IF}} = \frac{\boldsymbol{P}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{m}_0}{\|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{P}\|} = \frac{(\boldsymbol{I}_{N_B} - \boldsymbol{u}_0\boldsymbol{u}_0^{\mathrm{H}})\boldsymbol{m}_0}{\|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{P}\|}$$

and

$$\begin{aligned}\left\|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{P}\right\|^2 &= \mathrm{tr}(\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{P}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{m}_0) \\ &= \|\boldsymbol{m}_0\|^2 - \left|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{u}_0\right|^2 = 1 - R_B^2\end{aligned}$$

where

$$R_B = \frac{\left|\boldsymbol{b}_{BA}\boldsymbol{b}_{BB}^{\mathrm{H}}\right|}{\|\boldsymbol{b}_{BA}\|\,\|\boldsymbol{b}_{BB}\|}$$

So

$$\left|\boldsymbol{b}_{BB}\boldsymbol{v}_B^{\mathrm{IF}}\right|^2_{\max} = \|\boldsymbol{b}_{BB}\|^2(1 - R_B^2)$$

and

$$\begin{aligned}\boldsymbol{v}_B^{\mathrm{IF}} &= \frac{(\boldsymbol{I}_{N_B} - \boldsymbol{u}_0\boldsymbol{u}_0^{\mathrm{H}})\boldsymbol{m}_0}{\sqrt{(1 - R_B^2)}} \\ &= \left[\boldsymbol{I}_{N_B} - \frac{\boldsymbol{b}_{BA}^{\mathrm{H}}\boldsymbol{b}_{BA}}{\|\boldsymbol{b}_{BA}\|^2}\right]\frac{\boldsymbol{b}_{BB}^{\mathrm{H}}}{\|\boldsymbol{b}_{BB}\|\sqrt{(1 - R_B^2)}}\end{aligned}$$

□

9.2 Proof of Theorem 2

Proof: It is clear that the optimal precoding vector $\boldsymbol{v}_B$ can be written two parts, the no interference component proportional to the optimal IF precoding vector $\boldsymbol{v}_B^{\mathrm{IF}}$, and an interference causing component proportional to the vector $\boldsymbol{u}_0$, shown in (22)

$$\boldsymbol{v}_B^{\mathrm{IC}} = \alpha\boldsymbol{v}_B^{\mathrm{IF}} + \beta\boldsymbol{u}_0 \tag{22}$$

Let us examine the two constraints

$$\boldsymbol{v}_B^{\mathrm{H}}\boldsymbol{v}_B = (\alpha\boldsymbol{v}_B^{\mathrm{IF}} + \beta\boldsymbol{u}_0)^{\mathrm{H}}(\alpha\boldsymbol{v}_B^{\mathrm{IF}} + \beta\boldsymbol{u}_0) = |\alpha|^2 + |\beta|^2 = 1$$

$$r_{BA}^2P_B|\boldsymbol{b}_{BA}\boldsymbol{v}_B|^2 = r_{BA}^2P_B|\beta|^2\|\boldsymbol{b}_{BA}\|^2 \le F$$

then

$$\|\beta\|^2 \le \frac{F}{r_{BA}^2P_B\|\boldsymbol{b}_{BA}\|^2}$$

Moreover

$$\begin{aligned}|\boldsymbol{b}_{BB}\boldsymbol{v}_B|^2 &= \lambda_{BB}\left|\boldsymbol{v}_B^{\mathrm{H}}\boldsymbol{m}_0\right|^2 \\ &= \lambda_{BB}\left|(\alpha\boldsymbol{v}_B^{\mathrm{IC}} + \beta\boldsymbol{u}_0)^{\mathrm{H}}\boldsymbol{m}_0\right|^2 \\ &= \lambda_{BB}\left|\frac{\alpha(1 - \left\|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{u}_0\right\|^2)}{\sqrt{(1 - R_B^2)}} + \beta\boldsymbol{u}_0^{\mathrm{H}}\boldsymbol{m}_0\right|^2\end{aligned} \tag{23}$$

and

$$\left|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{u}_0\right|^2 = \left|\frac{\boldsymbol{b}_{BB}\boldsymbol{b}_{BA}^{\mathrm{H}}}{\|\boldsymbol{b}_{BB}^{\mathrm{H}}\|\,\|\boldsymbol{b}_{BA}\|}\right|^2 = R_B^2$$

Without losing generality, let $\alpha \in \mathbb{R}$, and in order to maximise the signal power of user B, define

$$\beta = \bar{\beta}\frac{\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{u}_0}{|\boldsymbol{m}_0^{\mathrm{H}}\boldsymbol{u}_0|} \text{ and } \bar{\beta} \in \mathbb{R}$$

Then (22) becomes

$$v_B^{IC} = \sqrt{1-\beta^2} v_B^{IF} + \beta \frac{m_0^H u_0}{\|m_0^H u_0\|} u_0$$
$$= \sqrt{1-\beta^2} v_B^{IF} + \beta \frac{b_{BB} b_{BA}^H}{R_B} b_{BA}^H \tag{24}$$

and (23) becomes

$$|b_{BB} v_B|^2 = \lambda_{BB} \left| \alpha \sqrt{1-R_B^2} + \beta R_B \right|^2$$
$$= \|b_{BB}\|^2 \left| \sqrt{(1-\beta^2)(1-R_B^2)} + \beta R_B \right|^2$$

Then if

$$R_B \leq \sqrt{\frac{F}{r_{BA}^2 P_B \|b_{BA}\|^2}}$$

we have

$$\beta = R_B$$

and

$$\left| b_{BB} v_B^{IC} \right|_{\max}^2 = \|b_{BB}\|^2$$

else

$$\beta = \sqrt{\frac{F}{r_{BA}^2 P_B \|b_{BA}\|^2}}$$

and

$$\left| b_{BB} v_B^{IC} \right|_{\max}^2 = \|b_{BB}\|^2 \left| \sqrt{(1-\beta^2)(1-R_B^2)} + \beta R_B \right|^2$$

□

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Multiple Antennas Selection for Linear Precoding MISO Cognitive Radio

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***Abstract*—Using multiple antennas in coexisting radio systems can cancel or control the co-channel interference (CCI), hence improve the overall spectrum efficiency. However, one of the drawbacks of such techniques is the hardware complexity. Antenna selection technology may reduce such costs while partly keeping the advantages of multiple antennas. In this paper, we focus on the downlink of a linear precoding multiple input single output (MISO) cognitive radio (CR) system and apply antenna selection techniques in the transmitter side of the secondary system. We discuss the optimal, maximum norm, and our proposed subset optimal selection strategy, which has a lower computational complexity and reduces feedback information compared to the optimal method. The simulation results show that our proposed methods achieve near optimal performance in terms of SNR.**

I. Introduction

Cognitive radio (CR) is one of emerging technologies that can allow reuse of the spectrum and improve the spectrum efficiency [1]. The main problem of such a technology is the co-channel interference (CCI). Multiple-antenna is one potential solution since it uses space diversity and can offer multiplexing gain and diversity gain for radio systems [2]. Some previous work [3] using multiple-antenna techniques in CR environments has been performed in scenarios where only the transmitter side employs multiple antennas. These linear approaches, including maximal ratio transmission (MRT), zero-forcing (ZF), optimal interference free (IF), and optimal interference-constrained (IC), are based on beamforming technologies, and can avoid or control the CCI, therefore improving the system performance. However, the main drawback of multiple antenna techniques is the cost of radio frequency (RF) chains, including low noise power amplifiers, gain control units, digital to analogue converters, and several filters, which are the major cost of a transmitter. Increasing the number of antennas will lead to a significant increase in the cost since each antenna requires a RF link.

Antenna selection can reduce the hardware complexity while keeping much of the benefit of multiple antennas [4]. The key idea behind this technology is using only a subset of available antennas to transmit or receive signals, therefore reducing the cost of the transmitters. Antenna selection has often been limited to the receiver side since transmitter antenna selection needs a feedback channel to obtain CSI. However, it does not increase the complexity for a CR system because the transmitters need to know the CSI to control the CCI and a feedback channel is therefore necessary. Furthermore, if the receivers can process part of the selection task, the required date rate of feedback channel can be decreased.

In this paper, we concentrate on the downlink of multiple input and single output cognitive radio (MISO-CR) systems with antenna selection techniques. The IF and IC algorithms are used in the secondary system to control CCI. We compare the optimal, maximum norm, and subset optimal selection strategy in term of signal power to noise ratio (SNR). Our proposed strategy, the subset optimal approach, achieves near optimal performance and greatly reduces the computational complexity and these points are confirmed by simulation results.

The rest of this paper is organized as follows. Section II describes the system model and basic assumptions. The antenna selection strategies are discussed in Section III. Simulation results comparing the performance of these strategies are in Section IV. Finally, we conclude our paper in Section V.

All boldface letters indicate vectors (lower case) and matrices (upper case). The $\mathbf{A}^T$, $\mathbf{A}^*$, and $\mathbf{A}^H$ are the transpose, conjugate, and conjugate transpose of $\mathbf{A}$. The $\|\mathbf{h}\| = (\sum h_i h_i^*)^{1/2}$, is the norm or length of vector $\mathbf{h}$, and $\mathrm{E}[\cdot]$ denotes the mathematical expectation.

II. System Model

Consider a typical point-to-point coexisting environment in which there are two independent radio systems using the overlapping spectrum, just as in Figure 1. For the sake of simplicity, we assume that the primary system (system A) is a single input and single output (SISO) radio system, and the signals of the primary system cannot be detected by the receiver of the secondary system, which could happen for example for an outdoor primary and indoor system. However, the secondary system might interfere with primary system users' receivers. The channel of the primary system is defined as $h \in \mathbb{C}$. For the secondary system (system B), we assume that there are N antennas but $n (2 \leq n \leq N)$ RF chains in the transmitter side. The data channel and the interference channel for the secondary system are $\mathbf{h}_D = [h_{D1}, h_{D2}, \cdots, h_{DN}]$ and $\mathbf{h}_I = [h_{I1}, h_{I2}, \cdots, h_{IN}]$, where $h_{Dj}, h_{Ij} \in \mathbb{C}$. When transmitting signals, only n antennas are used among the N possible antennas. There are a total of $K = \binom{N}{n}$ possible antenna selections, and define the set of all possible selections as $\Omega = \{\omega_1, \omega_2, \cdots, \omega_K\}$. Without loss of generality, we define in each possible selection the chosen antenna indices

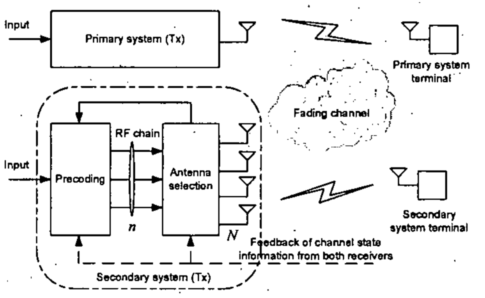

Fig. 1. Block diagram of MISO-CR system

to be in increasing order, then

$$\begin{aligned}\omega_1 &= \{1, 2, \cdots, n\}\\ \omega_2 &= \{1, 2, \cdots, n-1, n+1\}\\ \omega_K &= \{N-n+1, \cdots, N\}.\end{aligned}$$

If the subset ω_j is selected, define the channel as $\mathbf{h}^j$.

The received signals y_A and y_B for the nth sample time can be written as

$$\begin{aligned}y_A(n) &= h(n)s_A(n) + r\mathbf{h}_I^j(n)\mathbf{v}_B(n)s_B(n) + z_A(n)\\ y_B(n) &= \mathbf{h}_D^j(n)\mathbf{v}_B(n)s_B(n) + z_B(n)\end{aligned}$$

where s_A and s_B are the input signals for the primary system and the secondary system respectively, and the transmitter power constraint for system A and system B are denoted by $\mathrm{E}\left[\|s_A\|^2\right] \le P_A$ and $\mathrm{E}\left[\|s_B\|^2\right] \le P_B$. The vector $\mathbf{v}_B \in \mathbb{C}^{n\times 1}$ is the precoding vector for the secondary system providing that at least two antennas are selected, and we assume that $\|\mathbf{v}_B\|^2 = 1$. The additional noises z_A and z_B are independent complex Gaussian random variables with zero mean. Their covariances are σ_A^2 and σ_B^2 respectively. The scalar r, which is defined as interference factor, can allow for interference reduction due to waveform design, frequency overlap, etc. Therefore, the received signal to interference and noise power ratio (SINR) of the primary system and the secondary system for antenna selection ω_j are:

$$\mathrm{SINR}_A^j = \frac{P_A\|h\|^2}{\sigma_A^2 + r^2P_B\|\mathbf{h}_I^j\mathbf{v}_B\|^2} \quad \mathrm{SINR}_B^j = \frac{P_B\|\mathbf{h}_D^j\mathbf{v}_B\|^2}{\sigma_B^2}. \tag{1}$$

In the following discussion, we assume that the channel elements are independent, identically distributed (i.i.d) complex Gaussian random variables with zero mean and unit covariance.

III. Multiple-Antenna Selection Strategies

Three multiple-antenna selection strategies, including optimal, maximum norm, and our proposed subset optimal strategy are discussed in this section. The received SNR of the secondary system is our criterion for optimization whilst satisfying the interference requirements of the primary system. Linear precoding methods, IF and IC are used for the secondary system. The IF and IC methods achieve optimal linear beamforming performance when no interference and controlled interference to the primary system is allowed. The precoding vectors for IF and IC are [3]:

$$\mathbf{v}_B^{IF} = \left[\mathbf{I}_n - \frac{\left(\mathbf{h}_I^j\right)^H\mathbf{h}_I^j}{\|\mathbf{h}_I^j\|^2}\right]\frac{\left(\mathbf{h}_D^j\right)^H}{\left\|\mathbf{h}_D^j\right\|\sqrt{1-\rho^2}}$$

$$\mathbf{v}_B^{IC} = \sqrt{1-\beta^2}\mathbf{v}_B^{IF} + \beta\frac{\mathbf{h}_D^j\left(\mathbf{h}_I^j\right)^H}{\rho}\left(\mathbf{h}_I^j\right)^H$$

and

$$\beta = \min\left(\rho, \sqrt{\frac{F}{r^2P_B\|\mathbf{h}_I^j\|^2}}\right)$$

where $\min(x, y)$ is the minimal value of x and y, and F is interference to the primary system. Their corresponding channel gains are [3]:

$$G_{IF} = \left(1-\rho^2\right)\|\mathbf{h}_D^j\|^2 \tag{2}$$

$$G_{IC} = \left(\sqrt{(1-\beta^2)(1-\rho^2)} + \beta\rho\right)^2\left\|\mathbf{h}_D^j\right\|^2 \tag{3}$$

where ρ is the correlation coefficient between the data and interference channel of the secondary system defined as [3]:

$$\rho = \frac{\|\mathbf{h}_I^j(\mathbf{h}_D^j)^H\|}{\|\mathbf{h}_I^j\|\|\mathbf{h}_D^j\|} \tag{4}$$

β is the interference coefficient chosen to obey the IC interference constraint. For a given β, the interference power to the primary system is $F = \beta^2\|\mathbf{h}_I^j\|^2P_Br^2$. For the sake of the simplicity, we unify the equation (2) and (3) to one equation as follows:

$$G = \alpha\left\|\mathbf{h}_D^j\right\|^2 \tag{5}$$

where the value of α depends on which beamforming method is used and is a function of β and ρ. Moreover, $\alpha \in [0, 1]$. The interference gain is defined as $I = \beta^2\|\mathbf{h}_I^j\|^2$.

A. Optimal Strategy

The basic idea of the optimal strategy is to explore all the possible combinations of antennas and select a combination which maximises the SNR of the secondary system subject to the interference constraint. This method can be expressed as:

$$\omega_{op} = \underset{\omega_j\in\Omega}{\arg\max}\, G. \tag{6}$$

As shown in equation (2), (3), and (5), the value of G not only relate to the norm of selected channel, but also the correlation coefficient of interference and data channel ρ and/or interference coefficient β. Although this method can achieve the best performance, it has high computational complexity. To select n out of N antennas with an optimal strategy, we need to compute $K = \binom{N}{n} = \frac{N!}{n!(N-n)!}$ values of G, which increases very quickly when N increases. Another drawback of the optimal strategy is the requirement of CSI. Since all the combinations need to be estimated, both the interference channel and data channel state information for all available transmit antennas is required.

The performance of the optimal strategy normally is difficult to obtain since the set of random variables of channel gain G for all antenna combinations are not mutually independent of each other. Each antenna will be included in several different combinations, therefore leading to the correlation between these combinations. Moreover, the common antennas for two arbitrary combinations are different and the number of common antennas may range from 0 to $n-1$, which lead to the different correlation values between the antenna subsets.

Theorem 1: Consider the scenario described in Section II. Select n antennas from N available antennas for the secondary system while IF is used. If the data channel elements and interference channel elements are i.i.d complex Gaussian random variables with zero mean and unit covariance, the cumulative distribution function (CDF) of G_{IF}^{op} satisfies:

$$F(x) \geq F_{G_{IF}^{op}}(x) \geq F_1(x) \tag{7}$$

and

$$F_1(x) = KF(x) + 1 - K \tag{8}$$

$$F(x) = \frac{\gamma(n-1,x)}{(n-2)!} \tag{9}$$

where $\gamma(a,x)$ is lower incomplete gamma function [5], defined as

$$\gamma(a,x) = \int_0^x t^{a-1}e^{-t}dt$$

and

$$K = \frac{N!}{n!(N-n)!}$$

Proof: Consider any selected combination ω_j , rewrite the equation (2) as

$$G_{IF}^j = \left\|\mathbf{h}_D^j\right\|^2 - \left\|\sum_{k\in\omega_j} \frac{h_{Ik}}{\|\mathbf{h}_I^j\|} h_{Dk}^*\right\|^2. \tag{10}$$

Define a $n \times n$ unitary matrix $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_n]$, and

$$\mathbf{a}_1 = \left(\frac{\mathbf{h}_I^j}{\left\|\mathbf{h}_I^j\right\|}\right)^T.$$

Further define the size n random vector $\mathbf{u} = [u_1, u_2, \cdots, u_n]^T$, and $\mathbf{u}$ is the linear transformation of $\left(\mathbf{h}_D^j\right)^H$ via $\mathbf{A}$. It means that

$$\mathbf{u} = \mathbf{A}\left(\mathbf{h}_D^j\right)^H \quad \text{and} \quad u_1 = \sum_{k\in\omega_j} \frac{h_{Ik}}{\left\|\mathbf{h}_I^j\right\|} h_{Dk}^*. \tag{11}$$

Since $\mathbf{A}$ is a unitary matrix, and the elements of $\mathbf{h}_D^j$ are i.i.d complex Gaussian random variables with zero mean and unit covariance, $u_1, u_2, \cdots, u_n$ also are i.i.d complex Gaussian random variables with zero mean and unit covariance [6]. Furthermore,

$$\|\mathbf{u}\|^2 = \mathbf{h}_D^j \mathbf{A}^H \mathbf{A}\left(\mathbf{h}_D^j\right)^H = \left\|\mathbf{h}_D^j\right\|^2. \tag{12}$$

Then equation (10) becomes:

$$G_{IF}^j = \|\mathbf{u}\|^2 - \|u_1\|^2 = \sum_{k=2}^{n} \|u_k\|^2. \tag{13}$$

Thus, $2G_{IF}^j \sim \chi^2(2n-2)$. So the CDF of channel gain for antenna combination ω_j is [7]:

$$F_{G_{IF}^j}(x) = \Pr\left\{2G_{IF}^j \leq 2x\right\} = \frac{\gamma(n-1,x)}{(n-2)!} \tag{14}$$

where $\gamma(a,x) = \int_0^x t^{a-1}e^{-t}dt$ is the lower incomplete gamma functions [5]. Therefore, channel gains for any antenna combination are identically distributed random variables and define their CDFs as $F(x) = F_{G_{IF}^j}(x)$.

Furthermore, the CDF of optimal antenna combination which maximises the data channel gain can be written as:

$$F_{G_{IF}^{op}}(x) = \Pr\left\{\max_{\omega_j\in\Omega}\left\{G_{IF}^j\right\} \leq x\right\} \tag{15}$$

Since random variables sequence G_{IF}^j are not fully independent, then [8]

$$F(x) \geq F_{G_{IF}^{op}}(x) \geq F_1(x) \tag{16}$$

where

$$F_1(x) = KF(x) + 1 - K \tag{17}$$

$$K = \frac{N!}{n!(N-n)!} \tag{18}$$

■

Theorem 1 supplies a lower bound and a upper bound of CDF of data channel gain for optimal strategy. However, the lower bound is not tight since only when $F(x) \geq 1 - \frac{1}{K}$, $F_1(x) \geq 0$. If a constraint is given to n, a tighter lower bound can be obtained.

Corollary 1: If $n = N-1$, then the cumulative distribution function (CDF) of G_{IF}^{op} satisfies:

$$F_{G_{IF}^{op}}(x) > \left[\frac{\gamma(n-1,x)}{(n-2)!}\right]^{n+1}. \tag{19}$$

And it is a tighter lower bound compared to Theorem 1.

Proof: Since $n = N-1$, then $K = \binom{N}{n} = N = n+1$. For each antenna combinations, there is only one antenna which has not been included. Moreover, for any two combinations, there are $n-1$ common antennas and one different antenna. Since the channels are i.i.d random variables, we can say that random variables sequence, data channel gains of IF for each antenna combination, are exchangeable. Therefore, according to equation (15) and [9], the cumulative distribution function (CDF) of G_{IF}^{op} satisfies:

$$F_{G_{IF}^{op}}(x) > F^N(x) = \left[\frac{\gamma(n-1,x)}{(n-2)!}\right]^{n+1} \tag{20}$$

Now let us prove it is a tighter lower bound compared to Theorem 1. Define

$$g(x) = F^K(x) - F_1(x) \tag{21}$$

then

$$\begin{aligned} g(x)|_{K=N} &= F^N(x) - [1 - N + NF(x)] \\ &= [1 - F(x)]\,[N - \sum_{k=1}^{N} F^{N-k}(x)] \end{aligned}$$

Since $F(x)$ is CDF of random variable, $F(x) \leq 1$. Then

$$N - \sum_{k=1}^{N} F^{N-k}(x) \geq 0.$$

Therefore,

$$g(x)|_{K=N} \geq 0. \tag{22}$$

So $F^K(x) \geq F_1(x)$ and $F^K(x)$ is a tighter lower bound compared to $F_1(x)$. ∎

B. Maximum Norm Strategy

The optimal strategy can achieve best performance but it has high computational complexity and requires full CSI. If we just consider one part of the channel gain, the computational complexity may be decreased. Just to consider the correlation is not a good idea. From equation (2) and (3), it is easy to see that the correlation ρ between selected interference channel and data channel should tend to zero if we would like to maximise the channel gain without considering the norm $\left\|\mathbf{h}_D^j\right\|$. However, to perform this process, we need to calculate the correlation for each possible combination and full CSI is required for the transmitter. Therefore, it has the same complexity as the optimal method, but obtain worse results. The other part is the norm of the selected data channel. To achieve the best performance without the correlation value we need maximise the norm $\left\|\mathbf{h}_D^j\right\|$. Selecting the antennas with the largest n amplitudes can ensure the largest norm. Moreover, this calculation can be performed at the receiver side and it only needs to feedback selected antenna indexes and corresponding channel information. Therefore the maximum norm strategy not only reduces the computational complexity significantly but also decreases the bandwidth requirements of the feedback channel. The maximum norm strategy is presented as:

$$\omega_{\text{max-norm}} = \arg\max_{\omega_j \in \Omega} \left\|\mathbf{h}_D^j\right\|^2. \tag{23}$$

For a given channel, define $h_D^{(1)}, h_D^{(2)}, \cdots, h_D^{(N)}$ as the ordered data channels, where $h_D^{(1)}$ has the largest amplitude and $h_D^{(N)}$ is the smallest one. Then the channel gain with maximum norm strategy should be

$$G = \alpha \sum_{k=1}^{n} \left\|h_D^{(k)}\right\|^2. \tag{24}$$

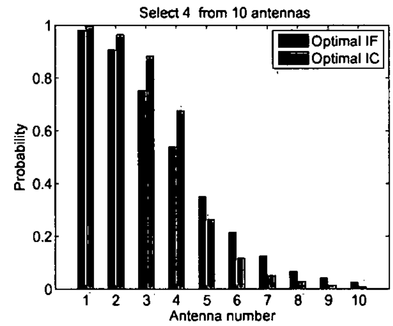

Fig. 2. Selection probability of the antenna with ordered channel gain for optimal strategy

Since only the norm part is considered, the SNR performance of the secondary system will be poorer than the optimal strategy. Now we discuss the average SNR received by the secondary system for IF precoding. Define θ be the angle between interference channel and data channel, and $\sin^2\theta = 1 - \rho^2$, where $\theta \in [0, \pi/2]$; and the maximum norm is $u = \sum_{k=1}^{n} \left\|h_D^{(k)}\right\|^2$. Because the angle and the amplitude are independent, the random variables θ and u are also independent random variables. Thus, from equation (1) and (2), the average SNR for the secondary system is

$$\mathrm{E}\left[\mathrm{SNR}_B\right] = \frac{P_B}{\sigma_B^2}\mathrm{E}\left[\sin^2\theta\right]\mathrm{E}\left[u\right] \tag{25}$$

From [10], the probability density function (PDF) of θ is $f_\theta(x) = 2(n-1)\sin^{2n-3}x\cos x$, where $x \in [0, \pi/2]$. Then

$$\mathrm{E}\left[\sin^2\theta\right] = \int_0^{\frac{\pi}{2}} \sin^2 x f_\theta(x)\,dx = 1 - \frac{1}{n}. \tag{26}$$

And according to [11], $\mathrm{E}[u] = n + \sum_{k=n+1}^{N} \frac{n}{k}$. Therefore, average SNR of the secondary system can be obtained from equation (26) and (25)

$$\mathrm{E}\left[\mathrm{SNR}_B\right] = \frac{P_B}{\sigma_B^2}\left[n - 1 + (n-1)\sum_{k=n+1}^{N}\frac{1}{k}\right] \tag{27}$$

where $n \geq 2$. By adding one more available antenna to N antennas, the SNR will increase by a factor $\frac{n-1}{N+1} \times 100\%$.

C. Subset Optimal Strategy

We discuss the optimal strategy and the maximum norm strategy in the previous subsections. The optimal strategy is not a practical method due to the computational complexity. Although the maximum norm method is easy to implement, it will lose some performance. The antenna selection probability according to amplitude for optimal selection strategy is shown in Figure 2, where the power of additive white noise for both

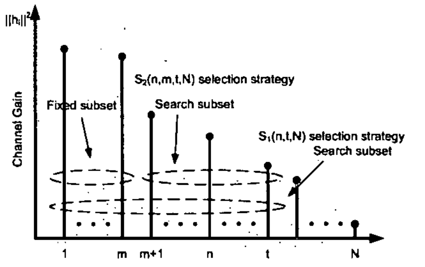

Fig. 3. Subset optimal antenna selection strategies $S_1(n,t,N)$ and $S_2(n,m,t,N)$

systems are 0dB and interference power constraint to the primary system $F \leq 0.2$ for IC. In this figure, we order antennas by decreasing norm, then plot the probabilities that they are selected by the optimal antenna selection method. The scenario is to select the best 4 antennas from 10 antennas with IF and IC precoding. It is clear that although the largest 4 antennas have a larger selection probabilities, the antennas with smaller norm sometimes are still selected. This difference of selection probabilities between optimal strategy and maximum norm strategy can explain why performance is lost for maximum norm strategy.

Now reconsider the optimal method. The reason for the high computational complexity is exploring all the possible combinations. If we decrease the number of elements, the computational complexity will definitely decrease. According to this analysis, we propose a subset optimal antenna selection strategy. This method has two steps:

1) Select a subset from all the available antennas according to a certain rule (maximum norm is used here);
2) Explore all possible antenna combinations among the selected subset to find the optimal solution;

The key point for this method is how to select the subset. From Figure 2, we can see that the larger the amplitude, the higher probability to be selected. Although sometimes antennas with the smaller amplitude are selected, the selection probabilities are very small. So if we do not consider these smallest-amplitude antennas, we should not lose a lot of performance. Therefore, we can use the amplitude as our subset selection criterion. This method is explained in Figure 3. Define the subset selection strategy $S_1(n,t,N)$, where N is the number of all available antennas, $t(N \geq t \geq n)$ is the size of the subset selected from the N available antennas (shown by an ellipse in Figure 3) and this subset includes the t largest amplitude antennas; n is the number of selected antennas, which is equal to the number of RF chains.

The computational complexity for $S_1(n,t,N)$ is $K_1 = \binom{t}{n}$. Increasing the size of subset t will increase the computational complexity. Since the process of selecting the subset can be performed in receiver side, t also specifies the amount of feedback information. The larger value of t, the more computation is required, the more feedback information is needed, but the better performance is achieved. If $t = N$, this method becomes the most complex optimal strategy; however if $t = n$, this method becomes maximum norm strategy.

To further decrease the computational complexity, we can use the subset optimal selection strategy $S_2(n,m,t,N)$, shown in Figure 3. We still select a subset from all available antennas, and the definitions of N, t, and n are the same as $S_1(n,t,N)$. The key idea for this strategy is that the largest $m(0 \leq m \leq n)$ antennas are fixed and definitely selected (the left ellipse in Figure 3) so we do not need fully explore the selected subset. We will not lose much performance because the probabilities of the antennas with largest amplitude tend to one, just as shown in Figure 2. Then the size of the subset we need to explore is $t - m$. Thus, the computational complexity for $S_2(n,m,t,N)$ is $K_2 = \binom{t-m}{n-m}$. Increasing t or decreasing m will lead to the improvement of performance at the cost of higher computational complexity. Another thing need to be noted that the required feedback information of $S_1(n,t,N)$ and $S_2(n,m,t,N)$ are same, and both of them are related to the subset size t.

IV. Simulation & Results

The simulation results of these three antenna selection strategies are shown in this section. We assume that the power of the secondary system $P_B = 0dB$, interference factor $r = 1.0$, and the noise power $\sigma_A^2 = \sigma_B^2 = 1.0$. For IC beamforming, the interference constraint $F \leq 0.2$. There are a total of $N = 10$ antennas in the transmitter of the secondary system, and we compare the SNR performance of the secondary system for the optimal selection, maximum norm, and subset optimal selection strategies where the number of RF chain $n \in \{2, 3, \cdots, 7\}$. The results for IF and IC are shown in Figure 4 and Figure 5 respectively. It is clear that the maximum norm method achieves worst performance for both IC and IF precoding approaches. It is because this method does not consider the angle between the interference channel and the data channel. Our proposed subset methods perform closer to the optimal method. For IF precoding, considering only 2 more antennas than the RF chain size (i.e. $t = n + 2$) will lead to near to optimal SNR performance; for the IC approach, adding just 1 antenna than the RF chain size can almost achieve the optimal SNR performance.

The comparison of computational complexity, required feedback information, and performance for different multiple antenna selection strategies are listed in Table I. We assume that the optimal strategy achieves 100% SNR performance, and the other strategies are compared to the optimal value. The computational complexity is the number of subset metrics G to be computed, and the required CSI indicates the number of complex channel realizations. For example, select 4 antennas from 10 available antennas, and it means there are $K = \binom{10}{4} = 210$ antenna combinations. If optimal selection strategy is used, 210 data channel gains G should be calculated

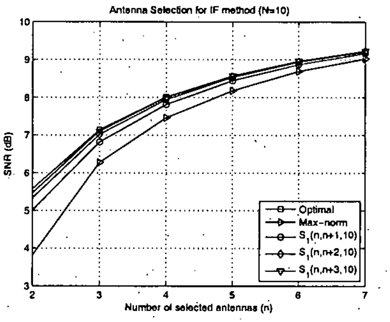

Fig. 4. Comparison of SNR for antenna selection with IF method

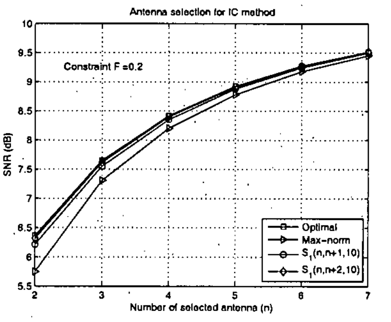

Fig. 5. Comparison of SNR for antenna selection with IC method

to obtain the maximum one. Furthermore, to calculate these channel gains in the transmitter of the secondary system, all the data CSI and interference CSI for 10 antennas should be feedback by users of the primary system and secondary system respectively. Therefore, the required CSI is 20 (the number of complex values).

We can see from this table that the maximum norm strategy can achieve acceptable performance when the IC approach is used for the secondary system; the proposed strategies, $S_1(4,6,10)$ and $S_2(4,2,6,10)$ with very low computational complexity and little CSI requirements achieve almost optimal performance, especially when IC beamforming is used.

V. CONCLUSION

Cognitive radio can reuse the frequency, therefore increasing the spectrum efficiency. Multiple antenna technologies have been used in coexisting environments to cancel the CCI. However,the cost of the RF chain, and the requirements of the feedback channel have limited the application of multiple antennas. In this paper, we apply antenna selection algorithms to solve these problems. The linear precoding methods IF and IC are used. After presenting the optimal selection strategy and the maximum norm selection strategy, we propose a subset optimal multiple antenna selection strategy. The key idea of our proposed method is to only explore a subset of all the available antennas based on a ranking metric, such as maximum norm. The simulation results show that our proposed method greatly reduces the computational complexity whilst achieving the near optimal performance.

TABLE I
COMPARISON OF ANTENNA SELECTION STRATEGIES $n = 4$, $N = 10$

	Computational complexity	Required CSI	IF SNR	IC SNR ($F \leq 0.2$)
Optimal	210	20	100%	100%
Max-norm	—	8	87.5%	95.1%
$S_1(4,6,10)$	15	12	97.8%	99.4%
$S_2(4,2,6,10)$	6	12	97.2%	99.2%

ACKNOWLEDGMENT

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SINGLE-ANTENNA SELECTION FOR MISO COGNITIVE RADIO

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Keywords: Multi-antenna, antenna selection, cognitive radio

Abstract

Cognitive radio with multiple antennas has promised significant improvements in spectrum efficiency. However the cost of radio frequency (RF) chains limits the application of multiple antennas. In this paper, we consider a scenario where the primary system can afford some interference and only one antenna is selected among N antennas in the secondary system. We first analyse the performance of the minimum interference selection, the maximum data power, and the maximum sum capacity strategies. Then we propose a maximum signal power to leak interference power ratio (SLIR) strategy, which considers both the leaked interference to the primary system and the data power simultaneously. The simulation results show that our proposed method achieves a better trade-off between two coexisting systems and can obtain good overall capacity performance.

1 Introduction

Cognitive radio (CR) is one of emerging technologies that can allow reuse of the spectrum and improve the spectrum efficiency [1,2]. The main problems of such a technology in the physical layer are how to cancel the co-channel interference (CCI) to avoid affecting the quality of service (QoS) of the primary system. Multiple antennas is one potential technology since it uses a new degree of freedom - the space dimension that can offer multiplexing gain and diversity gain for radio systems [3]. Some previous work[4] using multiple antenna technologies in cognitive radio environments has been done in scenarios where only the transmitter side employs multiple antennas due to the cost and power consumption. These approaches can avoid or control the CCI, therefore improving the system performance.

However, one of the drawbacks of multiple antenna techniques is the cost of radio frequency (RF) chains. In a transmitter, a RF link usually includes low noise power amplifiers, gain control units, analogue to digital converters, and several filters. These components are a significant part of the total cost of a transmitter. For each antenna, a RF link is required. Hence, increasing the number of antennas will lead to a significant increase in the transmitter cost. This problem may be solved via antenna selection technologies. Antenna selection can reduce the hardware complexity whilst keeping much of the benefit of multiple antennas [5, 6]. The key idea behind this technology is using only a subset of available antennas to transmit or receive signals. The RF chains adaptively switch them to a subset of all available antennas according to a specific optimization criterion for a given channel realization.

In this paper, we concentrate on the downlink of multiple input and single output cognitive radio (MISO-CR) systems with antenna selection technologies. To reduce the cost of the transmitter as much as possible, the scenario where only one antenna is selected in the transmitter side of the secondary system is considered. Selecting multiple antennas in MISO-CR with linear precoding is discussed in [7]. Four selection strategies, including maximum data gain, minimum interference, maximum sum capacity, and the maximum signal power to leak interference power ratio (SLIR), which is proposed here, are analysed and discussed. The results show that our proposed SLIR approach achieves a better performance trade-off between two coexisting systems and improves the spectrum efficiency.

The rest of this paper is organized as follows. We first describe the cognitive radio system model and basic assumptions in Section 2. Then the single antenna selection approaches according to different situations are discussed in Section 3. The numerical results are given in Section 4, and finally, we conclude our report in Section 5.

2 System model

Consider a typical point-to-point coexisting environment in which there are two independent radio systems using the overlapping spectrum, just as in Figure 1. For the sake of simplicity, we assume that the primary system (system A) is a single input and single output (SISO) radio system. According to the basic definitions of CR, we assume that there is no interference from the primary system to the secondary system. The channel of the primary system is defined as $h \in \mathbb{C}$. For the secondary system (system B), we assume that there are N ($N \geq 2$) antennas but only one RF chain in the transmitter of the secondary system. The data channel and the interference channel for the secondary system are $\mathbf{h}_D = [h_{D1}, h_{D2}, \cdots, h_{DN}]$ and $\mathbf{h}_I = [h_{I1}, h_{I2}, \cdots, h_{IN}]$, where $h_{Dj}, h_{Ij} \in \mathbb{C}$. When transmitting signals, only one

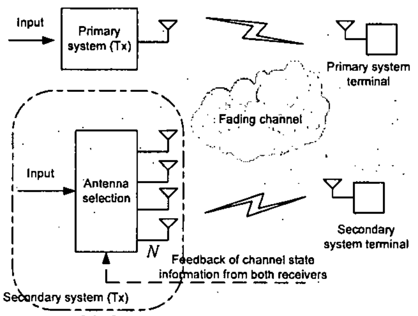

Figure 1 MISO CR with secondary system antenna selection

antenna is selected and used among the N possible antennas according to the CSI. Then when antenna j is selected, the received signals for both radio systems are:

$$y_A(n) = h(n)s_A(n) + rh_{Ij}(n)s_B(n) + z_A(n) \quad (1)$$

$$y_B(n) = h_{Dj}(n)s_B(n) + z_B(n) \quad (2)$$

where $s_A(n)$ and $s_B(n)$ are the input signals for the primary system A and the secondary system B respectively. The total transmitter power constraints for system A and system B are denoted by $\mathrm{E}[\|s_A\|^2] \le P_A$ and $\mathrm{E}[\|s_B\|^2] \le P_B$; the scalars $z_A(n)$ and $z_B(n)$ are independent complex Gaussian random variables with zero mean. Their covariances are σ_A^2 and σ_B^2 respectively; the scalar r, which is defined as interference factor, can allow for interference reduction due to waveform design, frequency overlap, etc. Therefore, the received signal to interference and noise power ratio (SINR) for the primary system and the secondary system are:

$$\mathrm{SINR}_A^j = \frac{P_A\|h\|^2}{\sigma_A^2 + r^2 P_B\|h_{Ij}\|^2} \text{ and } \mathrm{SINR}_B^j = \frac{P_B\|h_{Dj}\|^2}{\sigma_B^2} \quad (3)$$

In the following discussion, we also assume that the channel elements are independent, identically distributed (i.i.d) complex Gaussian random variables with zero mean and unit covariance.

3 Single antenna selection

The single-antenna selection strategy is the cheapest solution, since only one RF chain is needed in the transmitter side. However, the primary system has to suffer the CCI from the secondary system since it is unavoidable. We discuss four single-antenna selection strategies in follows.

3.1 Minimum interference strategy

To have little negative influence to the primary system, the secondary system should ensure the CCI is as small as possible. The obvious method is to select the antenna that has the minimum interference gain. It can be presented as follows:

$$j_{\text{min-Int}} = \arg\min_{j\in\{1,\cdots,N\}} \|h_{Ij}\|^2 \quad (4)$$

The key point behind this idea is to exploit channel fading to reduce the interference. Normally channel fading is a disadvantage of the radio system, and it sometimes may lead very lower SNR in the receiver side because of the deep fading caused by multiple reflections. Providing that multiple antennas are used and the channels are independent, we have larger probability to find a weaker interference channel than single antenna system therefore decreasing the interference and increasing the SINR of the coexisting radio system. We call the decrease of interference caused by independence of fading channel as "interference diversity gain".

Lemma 1: Let $\mathbf{h}_I$ be the interference channel whose elements are i.i.d complex Gaussian distributed with zero mean and unit covariance, and interference channel gain $g_j = \|h_{Ij}\|^2$ for $j \in \{1, 2, \cdots, N\}$. Define the random variable $g_{\min} = \min g_j$, for any $j \in \{1, 2, \cdots, N\}$; then the cumulative distribution function (CDF) $F_{g_{\min}}(x)$ and probability density function (PDF) $f_{g_{\min}}(x)$ of $g_{\min}$ are:

$$F_{g_{\min}}(x) = 1 - e^{-Nx} \quad f_{g_{\min}}(x) = Ne^{-Nx} \quad (5)$$

where $x \ge 0$; for $x < 0$, $F_{g_{\min}}(x) = f_{g_{\min}}(x) = 0$.

Proof. See [7].

The average interference power $G_{\min}$ is:

$$G_{\min} = \mathrm{E}[g_{\min}] = \int_0^{+\infty} x f_{g_{\min}}(x)\,dx = 1/N \quad (6)$$

From equation (6), it is clear that increasing the number of available antennas N will lead to the decrease of the interference to the primary system. Doubling the number of available antennas will achieve a 3dB decrease in interference on average. For an extreme situation, when $N \to \infty$, the average interference power $G_{\min} \to 0$. Next we consider the SINR for both systems.

Theorem 1: for the coexisting radio system defined in section 2, when the minimum interference power strategy of antenna selection is used, the CDFs of the SINR for both systems are:

$$F_{\mathrm{SINR}_A}(x) = 1 - \frac{P_A N e^{-\frac{\sigma_A^2}{P_A}x}}{P_A N + r^2 P_B x} \quad F_{\mathrm{SNR}_B}(x) = 1 - e^{\frac{-x\sigma_B^2}{P_B}} \quad (7)$$

and the mathematical expectations for $\mathrm{SINR_A}$ and $\mathrm{SNR_B}$ are:

$$\mathrm{E}[\mathrm{SINR}_A] = \frac{NP_A}{r^2 P_B} e^{\frac{N\sigma_A^2}{r^2 P_B}} \mathrm{E}_1\left(\frac{N\sigma_A^2}{r^2 P_B}\right) \quad (8)$$

$$\mathrm{E}\left[\mathrm{SNR}_B\right] = \frac{P_B}{\sigma_B^2} \qquad (9)$$

where $\mathrm{E}_1(x)$ is the order one exponential integral function [8]. For any $x \geq 0$, $\mathrm{E}_1(x)$ is defined as:

$$\mathrm{E}_1(x) = \int_x^{\infty} \frac{e^{-t}}{t} dt = -\gamma - \ln(x) - \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n!n}$$

where γ is Euler constant, which is 0.5772156649.

Proof: see [7].

The minimum interference strategy has minimum influence to the primary system, and it only needs the primary user to feedback which antenna has the least interference. However, the secondary system behaves like a single-input and single-output system, and it will not obtain any benefit from the multiple antennas.

3.2 Maximum data gain strategy

The minimum interference strategy maximises the performance of the primary system in terms of SINR while the secondary system loses all potential diversity gain. Sometimes the QoS of the primary system is not affected by the CCI caused by the secondary system due to the use of higher signal power or a very small interference factor r. In such situations, the antenna selection is not used to reduce the interference, but to increase the performance of the secondary system. The antenna selection idea has been illustrated in [10], and we apply it to cognitive radio environments. We can select the antenna with the strongest channel gain, called the maximum data gain strategy, which presented as follows:

$$J_{\max} = \arg\max_{J\in\{1,\cdots,N\}} \left\|h_{DJ}\right\|^2 \qquad (10)$$

Define $g_J = \left\|h_{lJ}\right\|^2$, $g_{\max} = \max g_J$ for any $J \in \{1,2,\cdots,N\}$.

If the elements of data channel $\mathbf{h}_D$ are i.i.d complex Gaussian distributed with zero mean and unit covariance, the cumulative distribution function $F_{g_{\max}}(x)$ and probability density function $f_{g_{\max}}(x)$ of $g_{\max}$ are [9] (pp.337):

$$F_{\max}(x) = \left(1-e^{-x}\right)^N \qquad (11)$$

$$f_{\max}(x) = N\left(1-e^{-x}\right)^{N-1} e^{-x} \qquad (12)$$

where $x \geq 0$. Then the data selection gain $G_{\max}$ is [9]:

$$G_{\max} = \int_0^{\infty} xN\left(1-e^{-x}\right)^{N-1} e^{-x} dx = \sum_{k=1}^{N} \frac{1}{k} \qquad (13)$$

From equation (13), it is clear that we can obtain more channel gain via increasing the number of available antennas N. The larger N is, the smaller the additional benefit we obtain from adding one more antennas. If we let $N \to \infty$, we have $G_{\max} \to \infty$ because the harmonic series is unbounded [8]. From the equation (11) and (13), the CDF of SINR and ergodic SINR for the secondary system are:

$$F_{\mathrm{SNR}_B}(x) = (1-e^{-\frac{x\sigma_B^2}{P_B}})^N \text{ and } \mathrm{E}\left[\mathrm{SNR}_B\right] = \frac{P_B}{\sigma_B^2}\sum_{k=1}^{N}\frac{1}{k} \qquad (14)$$

Furthermore, since the interference channels and data channels are independent and the elements of interference channel are i.i.d random variables, selecting the antenna with maximum data channel gain has no influence on the interference channel. So for the interference channel, it is equal to random selection, which has same effect as $N=1$. Then the CDF of SINR and ergodic SINR for the primary system can be obtained from equation (7) and (8):

$$F_{\mathrm{SINR}_A}(x) = 1 - \frac{P_A e^{-\frac{\sigma_A^2}{P_A}x}}{P_A + r^2 P_B x} \qquad (15)$$

$$\mathrm{E}\left[\mathrm{SINR}_A\right] = \frac{P_A}{r^2 P_B} e^{\frac{\sigma_A^2}{r^2 P_B}} \mathrm{E}_1\left(\frac{\sigma_A^2}{r^2 P_B}\right) \qquad (16)$$

It is easy to understand that this method will decrease the performance of the primary system when compared to the minimum interference strategy. Only by decreasing the transmitter power of the secondary system can improve the SINR performance of the primary system.

3.3 Maximum sum capacity strategy

We considered two kinds of extreme situations in previous subsections; one is the best for the primary system and the other for the secondary system. However, our aim is to achieve the maximum spectrum efficiency and find the best trade-off between two coexisting radio systems. Therefore, the maximum sum throughput strategy, which selects the antenna that can achieve best overall sum capacity, can be used.

$$J_{\max\text{-}c} = \arg\max_{J\in\{1,\cdots,N\}} \log\left(1+\mathrm{SINR}_A^J\right) + \log\left(1+\mathrm{SNR}_B^J\right) \qquad (17)$$

From equation (17), it is clear that to select the optimal antenna from N available antennas, the transmitter of the secondary system needs to compute all possible SINRs for both systems. This means that all the CSI should be known by the secondary system, including the CSI of the primary system. Moreover, the primary system and the secondary system have equal priorities. The channel, transmitter power, and noise in receive decide which system has better performance. However, in fact, the secondary system has high probability to achieve better performance because there is no co-channel interference (CCI) for the secondary system in CR and the primary system usually has to suffer serious CCI.

3.4 Maximum SLIR strategy

Three selection strategies have been discussed in previous sections. For the minimum interference strategy, although we ensure the primary system experiences less interference, the secondary system has not obtained any benefit from the use of multiple antennas. If we allow more interference to the primary system, it is possible that the performance of the secondary system could be greatly improved. For the maximum data gain strategy, it may greatly decrease the performance of the primary system, and this is usually not allowed in a cognitive radio environment. Moreover, although the maximum sum capacity strategy can achieve best overall performance, it might sometimes greatly degrade the performance of the primary system, just like maximum data strategy. More seriously, this method needs all the CSI from both systems, and it usually is very difficult to achieve in a practical cognitive radio system. So we need to find an antenna selection strategy that has little negative influence on the primary system while the secondary system may benefit from it, and then increase the total spectrum efficiency.

In here, we propose a maximum signal power to leak interference ratio (SLIR) strategy to solve this problem. The key idea is to select the antenna with maximum SLIR. Define the SLIR for antenna j as $K_j = \|h_{Dj}\|^2 / \|h_{Ij}\|^2$, where $j \in \{1,\cdots,N\}$. Then the maximum SLIR strategy can be presented as follows:

$$j_{\max\text{-SLIR}} = \arg\max_{j\in\{1,\cdots,N\}} K_j \tag{18}$$

From the definition of SLIR, it is easy to understand that the interference channel and data channel are both considered, and the interference is more important than data channel gain. It has a higher probability to select an antenna which leak lower interference than an antenna which has high data channel gain. This can ensure that the performance of the primary system has little negative influence. On the other hand, this method does also consider the data channel. If the antennas leak similar levels of interference power, the antenna with the highest data channel gain will be selected, and the performance of the performance of the secondary system is improved. In fact, this strategy allows a trade-off in performance between the two coexisting radio systems.

Unlike the minimum interference and maximum data gain strategies, this method needs to compute all the maximum SLIR in transmitter side. Therefore, the receivers have to feedback the CSI of all available antennas for the secondary system.

4 Numerical results

In this section, simulations are used to prove and verity our analytical results and discussion. We compare the capacity performance of four single-antenna selection strategies. Assume that the transmitter power of the primary system P_A is 10dB, and the number of available antennas for the secondary system N is 4. Define the interference factor r equals to 0.5, and assume there is no interference from the primary system to the secondary system. Then we compare the capacity performance of minimum interference strategy, maximum data power strategy, max signal power to leak interference ratio strategy, and the maximum sum capacity strategy.

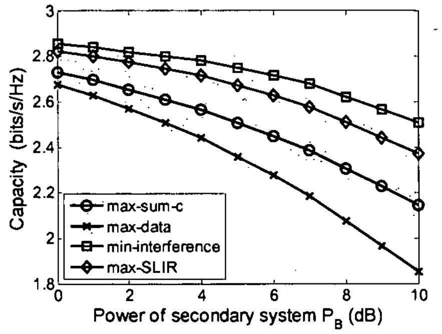

Figure 2 Capacity of primary system

The capacities of the primary system A are displayed in Figure 2. It is clear that the minimum interference method achieves best capacity performance. It is because for this strategy, space diversity is fully used to reduce the co-channel interference. As we expect, the maximum data power method obtains the worst performance for the primary system since it does not consider the CCI. For our proposed maximum SLIR strategy, it just loses a little capacity performance compared to the minimum interference strategy. For example, when the transmitter power of the secondary system $P_B = 8\text{dB}$, only 0.11 bits/s/Hz are lost compared to the minimum interference strategy. However, the maximum data power strategy loses 0.55 bits/s/Hz compared to the minimum interference strategy at the same situation.

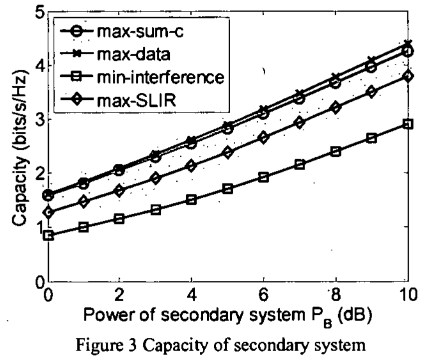

Figure 3 Capacity of secondary system

The capacity performance of the secondary system is described in Figure 3. Here, the maximum data power method obtains the best performance and the minimum interference method achieves the worst performance. The performance of our proposed method is better than the minimum interference method and worse than the maximum data methods. When $P_B = 8\text{dB}$, the maximum SLIR method outperforms the minimum interference method by 0.82 bits/s/Hz but loses 0.57 bits/s/Hz compared to the maximum data power methods.

Finally, we compare the sum capacities that are presented in Figure 4. Although the minimum interference method is the best method for system A, it will lose some sum capacity performance. The maximum data method achieves near optimal sum capacity because it greatly improves the performance of the secondary system and the interference factor is not large. Our proposed method improves the sum capacity compared to the minimum interference method whilst having little influence on the primary system. For $P_B = 8\text{dB}$, it loses 0.26 bits/s/Hz for sum capacity performance compared to the optimal sum capacity method, but the minimum interference method loses a further 0.96 bits/s/Hz. Therefore, it achieves a good trade-off between the two coexisting systems.

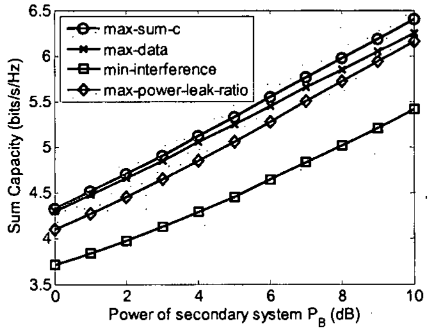

Figure 4 Sum capacity of both systems

5 Conclusion

Cognitive radio has received lots of attention because it provides a possibility to reuse radio frequencies and, therefore to increase the spectrum efficiency. Multiple antenna technologies have been used in coexisting environments to cancel the co-channel interference. However, the cost of the RF chain has limited the application of multiple antennas. In this paper, we apply antenna selection algorithms to solve this problem. A single-antenna selection scenario is considered.

The key point for the single-antenna selection scenario is that the co-channel interference can not be avoided. We first analyse the performance of the minimum interference selection strategy in which space diversity is used to reduce the CCI and the maximum data power strategy which achieves the best performance for the secondary system however can degrade the primary system. Then we propose a maximum signal power to leak interference power ratio (SLIR) strategy, which considers the leaked interference and data power at same time. The simulation results show that our proposed method achieves a better trade-off between two coexisting systems and obtain good overall capacity performance.

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MISO Aided Linear Interference Mitigation for Cognitive Radio

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Abstract—**Cognitive radio has attracted much attention due to the lack of spectrum resource and low usage statistics of existing spectrum allocations. Interference suppression and cancellation are seen as key technologies for enabling coexisting systems, and the application of multiple antennas might be one solution for this problem. This paper addresses linear vector precoding techniques for multiple input single output (MISO) cognitive radio systems. The optimal linear vector precoding algorithms for the secondary system in the sense of minimum mean squared error (MMSE) are proposed with reasonable constraints. Then we compare and analyze these approaches under different channel assumptions. The simulation results show that the proposed precoding algorithm can maximize the utilization of multiple antennas and greatly improve the system performance.**

Index Terms—**multi-input and single-output (MISO), interference cancellation, cognitive radio**

I. Introduction

Cognitive radio (CR), proposed in 2000 by Joe Mitola [1], has attached much attention due to the lack of radio spectrum resources and the low usage statistics of existing spectrum allocations [2] [3]. The key idea of cognitive radio is the coexistence of multiple radio systems in overlapping frequency or/and time slots and interference mitigation is the key factor. Multiple antenna technologies may be a potential solution for mitigating interference in coexisting environments. Generally, they can supply multiplexing gain, diversity gain, and co-channel interference suppression to the wireless system because of the independent channel fading between different pairs of antennas [4] [5].

Consider the downlink of coexisting environments with multiple antennas in base station side. The optimal precoding approach is dirty paper coding (DPC) [6] when the two systems can share the transmitted signals. However, the particular challenge of the cognitive radio is that both the transmitters and receivers are distributed and may be unable to coordinate with each other. Thus we consider approaches which do not need knowledge of the other system's transmitted signals. The maximum ratio transmission (MRT) method in [7] maximizes the received signal-to-noise ratio, but it does not consider the interference to the other radio system and therefore degrades its performance. The zero-forcing (ZF) method, which comes from multiple input and multiple output multi-user detection (MIMO-MUD) techniques [8], perfectly mitigates the interference to other radio systems. However, it may degrade the power of desired signals and lose some of the diversity gain of the channel.

The authors of this paper are among the core members of the Mobile VCE working group.

In this paper, our focus is on finding a precoding solution for the downlink of coexisting environments. We discuss single base station approaches. Firstly we present the optimal interference free (IF) approach, which is equivalent to TxZF in [9]; then interference constrained (IC) linear precoding algorithm is proposed when the primary system can suffer controlled co-channel interference (CCI). Moreover, we compare these approaches under different channel assumptions via analyses and simulations.

The rest of this paper is organized as follows. Section II describes the system model and basic assumptions of this paper. The known and proposed precoding approaches for a single system are presented in Section III. Simulation results comparing the algorithms under various conditions are given in Section IV, and finally, we conclude our paper in Section V.

All boldface letters indicate vectors (lower case) and matrices (upper case). The Tr(**A**), λ(**A**), Rank(**A**), and $\mathbf{A}^H$ are the trace, the eigenvalue, rank, and conjugate transpose of **A**. The $\|\mathbf{h}\| = (\sum h_i h_i^*)^{1/2}$, is the norm or length of vector **h**, and E[$\cdot$] denotes the mathematical expectation.

II. System Models

A block diagram of the multiple-input and single-output cognitive radio (MISO-CR) system for downlink transmission is shown in Fig. 1. Suppose that there are two radio systems, system A and system B. System A is the primary system and system B is the secondary system which tries to share the spectrum of system A. The base stations of system A and system B have N_A and N_B antennas respectively. We assume that $N_A \geq 2$ and $N_B \geq 2$. In the transmitters, the messages of system A and system B, s_A and $s_B \in \mathbb{C}$, are multiplied by the vectors $\mathbf{v}_A \in \mathbb{C}^{N_A \times 1}$ and $\mathbf{v}_B \in \mathbb{C}^{N_B \times 1}$, then transmitted over the frequency non-selective radio channels.

The received signals y_A and y_B are:

$$\begin{aligned} y_A &= \mathbf{h}_{AA}\mathbf{v}_A s_A + r_{BA}\mathbf{h}_{BA}\mathbf{v}_B s_B + z_A & (1) \\ y_B &= \underbrace{\mathbf{h}_{BB}\mathbf{v}_B s_B}_{\mathrm{I}} + \underbrace{r_{AB}\mathbf{h}_{AB}\mathbf{v}_A s_A}_{\mathrm{II}} + \underbrace{z_B}_{\mathrm{III}} & (2) \end{aligned}$$

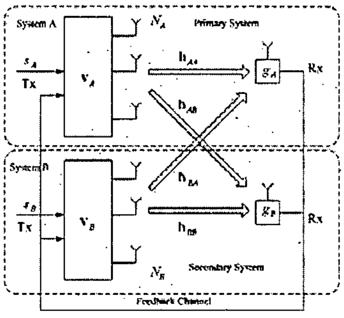

Fig. 1. Block diagram of multiple-input single-output cognitive radio system

where part I is the desired signal, part II is the co-channel interference (CCI), and part III is the additive noise. The row vectors $\mathbf{h}_{AA} \in \mathbb{C}^{1\times N_A}$, $\mathbf{h}_{AB} \in \mathbb{C}^{1\times N_A}$, $\mathbf{h}_{BB} \in \mathbb{C}^{1\times N_B}$, and $\mathbf{h}_{BA} \in \mathbb{C}^{1\times N_B}$ are channel vectors, whose elements are defined for different channel models. Moreover, define $\mathbf{h}_{AA}$, $\mathbf{h}_{BB}$ as message channels, and $\mathbf{h}_{AB}$, $\mathbf{h}_{BA}$ are interference channels with scaling channel weight r_{AB} and r_{BA} respectively. The noise term z_A and z_B are independent, identically distributed (i.i.d.) complex Gaussian random variables with zero mean. Their covariances are σ_A^2 and σ_B^2 respectively.

In the receivers, the received signals are multiplied by the complex scalars g_A and g_B respectively. The receivers also estimate their corresponding data channel and interference channel information, and according to different applications, feedback this channel information to the transmitters, as shown in Fig. 1.

We assume that the average transmit powers are fixed

$$\begin{aligned} \mathrm{E}\left[\|\mathbf{v}_A s_A\|^2\right] &= \mathrm{E}\left[\|s_A\|^2\right]\mathrm{Tr}\left(\mathbf{v}_A\mathbf{v}_A^H\right) = P_A \\ \mathrm{E}\left[\|\mathbf{v}_B s_B\|^2\right] &= \mathrm{E}\left[\|s_B\|^2\right]\mathrm{Tr}\left(\mathbf{v}_B\mathbf{v}_B^H\right) = P_B. \end{aligned}$$

Without loss of generality, define

$$\mathrm{E}\left[\|s_A\|^2\right] = P_A \text{ and } \mathrm{E}\left[\|s_B\|^2\right] = P_B$$

then

$$\mathrm{Tr}(\mathbf{v}_A\mathbf{v}_A^H) = \mathrm{Tr}(\mathbf{v}_B\mathbf{v}_B^H) = 1.$$

The received signal-to-interference-noise ratio (SINR) for system B is

$$\mathrm{SINR}_B = \frac{P_B\mathbf{v}_B^H\mathbf{h}_{BB}^H\mathbf{h}_{BB}\mathbf{v}_B}{r_{AB}^2 P_A\mathbf{v}_A^H\mathbf{h}_{AB}^H\mathbf{h}_{AB}\mathbf{v}_A + \sigma_B^2}. \quad (3)$$

The normalized mean squared error for system B is defined as:

$$\mathrm{MSE}_B^{nr} = \frac{\mathrm{E}\left[\|s_B - g_B y_B\|^2\right]}{P_B}. \quad (4)$$

For the fixed precoding vector $\mathbf{v}_A$ and $\mathbf{v}_B$, the minimum normalized mean squared error (MMSE) for system B is

$$\mathrm{MMSE}_B^{nr} = \frac{r_{AB}^2 P_A\mathbf{v}_A^H\mathbf{h}_{AB}^H\mathbf{h}_{AB}\mathbf{v}_A + \sigma_B^2}{P_B\mathbf{v}_B^H\mathbf{h}_{BB}^H\mathbf{h}_{BB}\mathbf{v}_B + r_{AB}^2 P_A\mathbf{v}_A^H\mathbf{h}_{AB}^H\mathbf{h}_{AB}\mathbf{v}_A + \sigma_B^2}. \quad (5)$$

Since system A and system B are symmetrical, the equations for SINR_A, MSE_A^{nr}, and MMSE_A^{nr} are simular to those for system B.

III. Single System Algorithms

The single system algorithms are used when the two systems cannot exchange their channel state information. Each system independently precodes depending on their own channel state information for user A and B. In this section, we consider the precoding methods for system B (the secondary system) and fix the precoding vector for system A (the primary system). We usually assume that the base station only knows the information of its own data channel and the interference channel to the primary system through the feedback of information from the terminals.

A. Maximum Ration Transmission (MRT)

The maximum ration transmission method (MRT), also named TxMF [7] [9], maximizes the received desired signal's power subject to limited transmitter power. For system B, the solution of this approach is the conjugate transpose of the message channel with power limitation:

$$\mathbf{v}_B^{MRT} = \frac{\mathbf{h}_{BB}^H}{\|\mathbf{h}_{BB}\|}. \quad (6)$$

This method provides the best performance for system B, but without considering the interference to system A. The maximal diversity gain against the fading channel is obtained. In addition, only the message channel information for system B is needed. The main drawback of MRT in coexisting environments is that it may greatly degrade the performance of the primary system due to the interference coming from the secondary system.

B. Zero-forcing (ZF)

The zero-forcing (ZF) method [8] can perfectly cancel the interference to system A. The key idea of this method is to find a vector that is orthogonal to the interference channel, $\mathbf{h}_{BA}\mathbf{v}_B = 0$. Define the system B interference self-correlation matrix $\mathbf{F}_B = \mathbf{h}_{BA}^H\mathbf{h}_{BA}$, and then $\mathrm{Rank}(\mathbf{F}_B) = 1$. Thus, it has one non-zero eigenvalue and $N_B - 1$ zero eigenvalues, $\lambda(\mathbf{F}_B) = \{\lambda_{non-zero}, 0, \cdots, 0\}$, where $\lambda_{non-zero} = \|\mathbf{h}_{BA}\|^2$. The eigenvectors corresponding to the zero eigenvalues of the interference self-correlation matrix form the solution of the ZF method. The data channel information for system B is not needed for this approach. A direct solution of ZF is:

$$\mathbf{v}_B^{ZF} = \frac{\overline{\mathbf{v}}_B}{\|\overline{\mathbf{v}}_B\|} \quad (7)$$

and

$$\overline{\mathbf{v}}_B = \mathbf{v} - \frac{\mathbf{h}_{BA}\mathbf{v}}{\|\mathbf{h}_{BA}\|^2}\mathbf{h}_{BA}^H, \mathbf{v} \in \mathbb{C}^{N_B\times 1}, \|\overline{\mathbf{v}}_B\| \neq 0. \quad (8)$$

If there are more than two antennas in base station B, the solution to the ZF algorithm is not unique because there are more than one independent eigenvectors corresponding to zero eigenvalues. An arbitrary choice of the ZF vector will lead to the loss of some diversity gain due to not exploiting the data channel information.

C. Optimal Interference-free (IF)

The optimal precoding vector in the sense of causing no interference to the primary system will provide better performance than the ZF method. We select minimizing MSE as our performance criterion. The problem is described as follows:

$$v_B^{IF} = \arg\min_{\{v_B\}} \mathrm{MSE}_B^{nr}(v_B)$$

$$\text{s.t.: } \|\mathbf{h}_{BA}\mathbf{v}_B\|^2 = 0 \text{ and } \mathrm{Tr}(\mathbf{v}_B\mathbf{v}_B^H) = 1.$$

Fix the precoding vector $\mathbf{v}_A$, and from (5), minimizing MSE translates into maximizing the scalar $\|\mathbf{h}_{BB}\mathbf{v}_B\|^2$ with the above constraints.

Theorem 1: Let $\mathbf{h}_{BB}$ and $\mathbf{h}_{BA} \in \mathbb{C}^{1\times N_B}$, $N_B \geq 2$, be the message channel and the interference channel row vectors respectively. If they are not linearly dependent, i.e. $\|\mathbf{h}_{BA}\mathbf{h}_{BB}^H\| \neq \|\mathbf{h}_{BA}\|\|\mathbf{h}_{BB}\|$, the optimal precoding vector $\mathbf{v}_B$ maximizing the $\|\mathbf{h}_{BB}\mathbf{v}_B\|^2$ with constraints $\|\mathbf{h}_{BA}\mathbf{v}_B\|^2 = 0$ and $\mathbf{v}_B^H\mathbf{v}_B = 1$ is:

$$\mathbf{v}_B^{IF} = \left[\mathbf{I}_{N_B} - \frac{\mathbf{h}_{BA}^H\mathbf{h}_{BA}}{\|\mathbf{h}_{BA}\|^2}\right]\frac{\mathbf{h}_{BB}^H}{\|\mathbf{h}_{BB}\|\sqrt{(1-R_B^2)}} \tag{9}$$

and

$$\|\mathbf{h}_{BB}\mathbf{v}_B^{IF}\|^2_{\max} = \|\mathbf{h}_{BB}\|^2(1-R_B^2) \tag{10}$$

where R_B is the cross-correlation coefficient, defined as:

$$R_B = \frac{\|\mathbf{h}_{BA}\mathbf{h}_{BB}^H\|}{\|\mathbf{h}_{BA}\|\|\mathbf{h}_{BB}\|}. \tag{11}$$

Proof: See [9] and [10]. ■

D. Optimal Interference-constrained (IC)

Sometimes the primary system may be able to tolerate controlled levels of interference in a practical system. In this scenario, this headroom may be used to help the secondary system achieve better performance. Keeping this in mind, an interference constrained precoding algorithm is proposed. The problem is described as follows:

$$\mathbf{v}_B^{IC} = \arg\min_{\{\mathbf{v}_B\}} \mathrm{MSE}_B^{nr}(\mathbf{v}_B)$$

$$\text{s.t.: } r_{BA}^2 P_B\|\mathbf{h}_{BA}\mathbf{v}_B\|^2 \leq F \text{ and } \mathrm{Tr}(\mathbf{v}_B\mathbf{v}_B^H) = 1.$$

The only difference to the IF scheme in the previous subsection is the interference constraint, therefore this problem is also equal to maximizing the scalar $\|\mathbf{h}_{BB}\mathbf{v}_B\|^2$ with above constraints.

Theorem 2: Let $\mathbf{h}_{BB}$ and $\mathbf{h}_{BA} \in \mathbb{C}^{1\times N_B}$, $N_B \geq 2$, be the message channel and the interference channel row vectors respectively. If they are not linearly dependent, i.e. $\|\mathbf{h}_{BA}\mathbf{h}_{BB}^H\| \neq \|\mathbf{h}_{BA}\|\|\mathbf{h}_{BB}\|$, the optimal precoding vector $\mathbf{v}_B$ maximizing the $\|\mathbf{h}_{BB}\mathbf{v}_B\|^2$ with constraints $r_{BA}^2P_B\|\mathbf{h}_{BA}\mathbf{v}_B\|^2 \leq F$ and $\mathbf{v}_B^H\mathbf{v}_B = 1$ is:

$$\mathbf{v}_B^{IC} = \sqrt{1-\bar{\beta}^2}\mathbf{v}_B^{IF} + \bar{\beta}\frac{\mathbf{h}_{BB}\mathbf{h}_{BA}^H}{R_B}\mathbf{h}_{BA}^H \tag{12}$$

where

$$\bar{\beta} \in \mathbb{R} \text{ and } 0 \leq \bar{\beta} \leq R_B.$$

If

$$\sqrt{\frac{F}{r_{BA}^2P_B\|\mathbf{h}_{BA}\|^2}} \geq R_B \tag{13}$$

then

$$\|\mathbf{h}_{BB}\mathbf{v}_B^{IC}\|^2_{\max} = \|\mathbf{h}_{BB}\|^2 \text{ and } \bar{\beta} = R_B \tag{14}$$

else

$$\|\mathbf{h}_{BB}\mathbf{v}_B^{IC}\|^2_{\max} = \|\mathbf{h}_{BB}\|^2\|\sqrt{(1-\bar{\beta}^2)(1-R_B^2)} + \bar{\beta}R_B\|^2 \tag{15}$$

and

$$\bar{\beta} = \sqrt{\frac{F}{r_{BA}^2P_B\|\mathbf{h}_{BA}\|^2}}. \tag{16}$$

Proof: See [10]. ■

TABLE I
COMPARISON OF SINGLE SYSTEM APPROACHES

	Interference power to primary system	Message channel power of secondary system (G)	Required channel state information
MRT	$r_{BA}^2R_B^2\|\mathbf{h}_{BA}\|^2P_B$	$\|\mathbf{h}_{BB}\|^2P_B$	Message channel
ZF	Zero	$\leq G_{IF}$	Interference channel
IF	Zero	$(1-R_B^2)\|\mathbf{h}_{BB}\|^2P_B$	Message and interference channel
IC	$\leq F$	$G_{IF} \leq G_{IC} \leq G_{MRT}$	Message and interference channel

E. Comparison of Single System Approaches

We presented the single system precoding approaches, MRT, ZF, IF, and IC in the four previous sections. The comparison of these approaches is presented in Table I.

IV. SIMULATION & RESULTS

The Rayleigh fading channel is considered in our simulations. The elements of the channel vectors are i.i.d. complex Gaussian random variables with zero mean and unit covariance. We also assume the channels are slow-fading wireless channels with packet-based transmission and are quasi-static over one packet length. For one simulation, a total of 2×10^6 packets are sent, and there are 10 symbols in each packet. Both radio systems send uncoded QPSK signals, and the noise in the receivers is set to unity (0dB).

Assume that system A employs two antennas in its base station. The ZF scheme is used by system A and it will not cause interference to system B. The transmission power of system A is fixed to 10dB. The base station of System B has 4 antennas and variable transmission power. We compare the BER performance of two systems when different precoding approaches are used.

The simulation results of system B and system A are shown in Fig.2 and Fig.3 respectively. From these two figures, we can see that MRT achieves the best performance for system B but simultaneously causes the poorest performance for system A. This is because MRT maximizes the received signals for

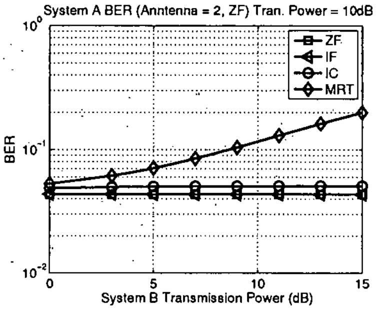

Fig. 2. BER of system A for ZF, IF, IC, and MRT under Rayleigh channel

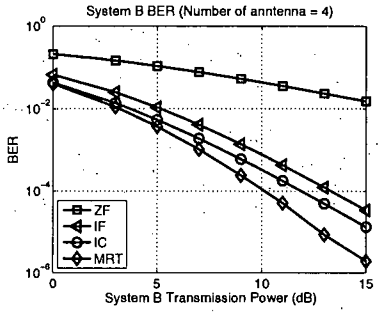

Fig. 3. BER of system B for ZF, IF, IC, and MRT under Rayleigh channel

system B (full diversity gain is obtained) and produces the maximum co-channel interference to system A which greatly degrades system A's SINR.

The ZF method achieves the poorest performance for system B because the arbitrary selection of an orthogonal vector loses all the diversity gain. The performance of the IF method is superior to ZF performance because the IF method selects the optimal orthogonal vector, and obtains the maximum achievable diversity gain. It is easy to understand the IF and ZF give the same performance for system A because they do not produce any interference to it.

The interference constraint for IC is 0.2, which is 20% of the noise power. The performance of IC for system B is in-between that of IF and MRT. This is because the IC utilizes part of interference vector to achieve higher SNR for system B. That also is why there is a gap between IC and IF for the performance of system A in Fig.3. Since the constraint is defined as the allowable interference power, which is directly proportional to the transmission power, the interference coefficient for IC $\bar{\beta}$ is inversely proportional to the transmission power. Higher transmission power corresponds to a lower coefficient $\bar{\beta}$ and vice versa. Moreover, a higher interference coefficient $\bar{\beta}$ leads to IC performance tending to that of MRT, and a lower interference coefficient leads to an IC solution tending towards that of the IF. It explains why the IC performance tends to that of MRT when transmission power is low, and tends to that of ZF when transmission power is high.

V. Conclusion

The linear vector precoding approaches for MISO-CR are considered in this paper. Based on the known algorithms, we present and prove the optimal interference free (IF) precoding vector, which is the optimal linear combination of ZF precoding vectors. When the primary system can afford some interference due to the good channel conditions, we propose an optimal interference-constrained (IC) precoding method, which utilizes this headroom to improve the performance of the secondary system. The comparison of these single system approaches gives us a clear view of their relationship. Simulation results show that the linear precoding algorithms using multiple antenna can mitigate or constrain the co-channel interference and improve the performance of coexisting system at the same time.

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